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Semi-local Cosmic Strings and the Cosmological Constant Problem

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Abstract

We study the cosmological constant problem in a three-dimensional $N = 2$ supergravity theory with gauge group $SU(2)_{global} \times U(1)_{local}$. The model we consider is known to admit string-like configurations, the so-called semi-local cosmic strings. We show that the stability of these solitonic solutions is provided by supersymmetry through the existence of a lower bound for the energy, even though the manifold of the Higgs vacuum does not contain non-contractible loops. Charged Killing spinors do exist over configurations that saturate the Bogomol'nyi bound, as a consequence of an Aharonov-Bohm-like effect. Nevertheless, there are no physical fermionic zero modes on these backgrounds. The exact vanishing of the cosmological constant does not imply, then, Bose-Fermi degeneracy. This provides a non-trivial example of the recent claim made by Witten on the vanishing of the cosmological constant in three dimensions without unphysical degeneracies.

The cosmological constant problem has longly survived the attempts made by physicists to distangle it. One of the most interesting views on the problem, in the context of three-dimensional supergravity, was recently given by Witten [1]: supersymmetry can ensure the exact vanishing of the cosmological constant without compelling bosons and fermions to be degenerate. Unbroken supercharges, which must be constructed from spinors that are supercovariantly constant at infinity are, in principle, ill-defined in 2+1 as a consequence of the asymptotically conical geometries produced by massive configurations [2]. Then, even when supersymmetry applies to the vacuum ensuring the vanishing of the cosmological constant, it is broken over the excited states. Though there is no indication that an analogous scenario can take place in four-dimensional space-times, some ways to extend this result were thereafter explored, using the ideas of strong coupling duality [3, 4] and S-duality [5].

The presumed non-existence of asymptotical Killing spinors in three-dimensional supergravities coupled to matter was recently shown to be overcome in presence of Nielsen-Olesen vortices [6, 7]. The geometric phase associated with the conical geometry results to be canceled by an Aharonov-Bohm phase produced by the vortex flux lines. Nevertheless, the would-be fermionic Nambu-Goldstone zero modes generated by the action of broken generators results to be non-normalizable, thus not entering in the physical Hilbert space [6]. Then, in spite of being possible to end with Killing spinors over certain solitonic backgrounds, there is no Bose-Fermi degeneracy. It is still possible to have a vanishing cosmological constant without implying such an unphysical degeneracy in the spectrum.

This result was recently shown to apply, under general hypothesis, to any $2+1$ dimensional system admitting topological solitons [8]. In all these models, the supercovariant derivative receives a contribution from the topological vector potential that allows the cancelation of the conical holonomy by a phase produced when surrounding the solitonic configuration. Thus, it can be shown that the existence of non-trivial supercovariantly constant spinors at infinity is guaranteed whenever massive field configurations saturate the corresponding Bogomol'nyi bound. This assertion is still valid even if the topological vector potential is an auxiliary field as in the CP^n model discussed in [8]. It seems to be a general result, however, that the fermionic zero modes receive an infrared divergent contribution from the gravitino transformation law related to the conical nature of the $2+1$ dimensional space-time.

Then, vanishing of the cosmological constant can still be thought of as a consequence of an underlying supersymmetry, without rendering bosons and fermions to be degenerate.

In the context of possible cosmic string scenarios, it is known that stable solutions can take place even if the topology looks trivial in a naïve sense: the manifold of minima for the potential energy does not contain non-contractible loops. This is the case, for example, of the so-called semi-local cosmic string introduced in [9], where stability is provided by the requirement that the gradient energy density falls off sufficiently fast at infinity. The stability of the static flat-space string solution, as well as the critical relation between coupling constants where it takes place, can be understood as coming from supersymmetry [10]: the semi-local cosmic string being thought of as a purely bosonic configuration of an $N = 2$ supersymmetric system. In the gravitationally coupled system, semi-local cosmic string and multi-string solutions were explicitely found and studied a few years ago [11].

In the present letter we would like to address the would-be supersymmetric nature of semi-local cosmic strings coupled to gravity, and its relation with the cosmological constant problem. We show that the semi-local cosmic string can be consistently embedded into an $N = 2$ supergravity theory whenever the critical relation between coupling constants takes place. Its stability is shown to be a consequence of the underlying $N = 2$ supersymmetry algebra. We study the existence of Killing spinors in a semi-local cosmic string background, mainly focusing on the case in which it saturates a Bogomol'nyi bound (though the system is, naively, non-topological). We discuss the relation between these solutions and Witten's claim about the vanishing of the cosmological constant without Bose-Fermi degeneracy in $2 + 1$ dimensions.

Let us write down the $SU(2)_{global} \times U(1)_{local}$ lagrangian density of our $2 + 1$ dimensional system,

$$V^{-1}\mathcal{L} = \frac{M_{pl}}{2}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(D_\mu\Phi)^\dagger(D^\mu\Phi) - \xi(\Phi^\dagger\Phi - v^2)^2 \ , \qquad (1)$$

which arises from the standard electroweak model minimally coupled to gravity by setting the $SU(2)$ gauge coupling constant to zero. V is the determinant of the dreibein, M_{pl} is the Planck mass and ξ is a real coupling constant. The Higgs field Φ is a complex doublet and the covariant derivative reflects the fact that we have only gauged the $U(1)$ factor of the gauge group, $D_\mu = \partial_\mu - ieA_\mu$. The Lagrangian (1) has a global $SU(2)$ symmetry as well as a local $U(1)$ invariance, under which the Higgs field changes as $\Phi \to e^{i\alpha}\Phi$. When the Higgs field acquires a definite vacuum expectation value the symmetry is broken to a global $U(1)$. The vacuum manifold is the three-sphere $|\Phi| = v$, which has no non-contractible loops. However, as the gradient energy density must fall off sufficiently fast asymptotically, fields at infinity owe to lie on a gauge orbit, that is, a circle lying on the three-sphere.

We are, as announced, interested in cosmic string configurations of the system described by (1) that could be understood as solutions of an $N = 2$ supergravity theory. We must then accomodate our matter fields into an $N = 2$ vector multiplet (A_μ, λ, S) and an $N = 2$ hypermultiplet (Φ, Ψ), transforming in the vector representation of $SU(2)$. We then accordingly enlarge the symmetry of our lagrangian density, coupling these multiplets to the Einstein supermultiplet, finding, after some tensor calculus,

$$\begin{aligned} V^{-1}\mathcal{L}_{N=2} &= \frac{M_{pl}}{2}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(D_\mu\Phi)^\dagger(D^\mu\Phi) + \frac{1}{2}\partial_\mu S\partial^\mu S \\ &+ \frac{e^2}{2}S^2\Phi^\dagger\Phi - \frac{e^2}{8}(\Phi^\dagger\Phi - v^2)^2 - \frac{V^{-1}}{2}\epsilon^{\rho\mu\sigma}\bar{\psi}_\rho\hat{\nabla}_\mu\psi_\sigma \\ &- \frac{1}{2}\bar{\lambda}\gamma^\mu\tilde{\nabla}_\mu\lambda - \frac{1}{2}\bar{\Psi}\gamma^\mu\tilde{\nabla}_\mu\Psi + V^{-1}L^{int}_{Fer} \ , \end{aligned} \qquad (2)$$

where the last term L^{int}_{Fer} includes several interaction terms involving fermions, whose explicit form will not be of interest for us. Note that $N = 2$ supersymmetry has forced a definite relation between the Higgs coupling constant and the electric charge

$$\xi = \frac{e^2}{8} \ , \qquad (3)$$

as it happens in the abelian Higgs model both in flat and curved space-time [7, 12]. Concerning the fermion derivatives in (2), they are given by the following expressions:

$$\hat{\nabla}_\mu\psi_\sigma = \left(\mathcal{D}_\mu + \frac{i}{4M_{pl}}J_\mu + ie\frac{v^2}{4M_{pl}}A_\mu\right)\psi_\sigma \ , \qquad (4)$$

$$\tilde{\nabla}_\mu\lambda = \left(\mathcal{D}_\mu - ie\frac{v^2}{4M_{pl}}A_\mu\right)\lambda \ , \qquad (5)$$

$$\tilde{\nabla}_\mu \Psi = \left(\mathcal{D}_\mu - ie \left(1 + \frac{v^2}{4M_{pl}} \right) A_\mu \right) \Psi \ , \tag{6}$$

where $\mathcal{D}_\mu$ is the general relativity covariant derivative acting on a spinor, and J_μ is the Higgs field current,

$$J_\mu = \frac{i}{2} \left((D_\mu \Phi)^\dagger \Phi - \Phi^\dagger (D_\mu \Phi) \right) \ . \tag{7}$$

Notice that all fermions have received a charge contribution of magnitude $\frac{ev^2}{4M_{pl}}$, as a consequence of the coupling of our system to $N = 2$ supergravity. This striking fact, related to the presence of a Fayet-Iliopoulos term in the supergravity lagrangian, induces a charged supersymmetric parameter. The relevance of this issue will become clear when studying the evasion of the no-go scenario posed by the asymptotically conical nature of the $2+1$ dimensional space-time. We will be mainly interested in purely bosonic field configurations of this system. We then introduce the following useful notation: given a functional Ξ depending both on bosonic and fermionic fields, we will use $\Xi|$ to refer to that functional evaluated in the purely bosonic background,

$$\Xi| \equiv \Xi|_{\lambda,\Psi,\psi_\mu=0} \ . \tag{8}$$

Under this condition, the only non-trivial supersymmetric transformations laws that leave invariant the lagrangian (2), are those corresponding to fermionic fields:

$$\delta\lambda| = \frac{1}{2} F_{\mu\nu}\sigma^{\mu\nu}\epsilon + \frac{ie}{4}(\Phi^\dagger\Phi - v^2)\epsilon + \gamma^\mu \partial_\mu S \epsilon \ , \tag{9}$$

$$\delta\Psi| = \frac{1}{2} \left(\not{D}\Phi + ieS\Phi \right) \epsilon \quad , \quad \delta\psi_\mu| = 2M_{pl}^{1/2} \hat{\nabla}_\mu \epsilon \ . \tag{10}$$

We are interested in field configurations describing semi-local cosmic strings that emerge from the system described by (1). Thus, we shall make at this point $S = 0$. Moreover, since Bogomol'nyi equations correspond to static configurations with $A_0 = 0$, we also impose these conditions (note that in this case $T_{0i}^{mat} = 0$).

The metric of a static spacetime can always be adapted to the time-like killing vector field $\frac{\partial}{\partial t}$, such that it is given by:

$$ds^2 = H^2 dt^2 - \Omega^2 dz d\bar{z} \ , \tag{11}$$

where we have written the metric on the surface Π, orthogonal everywhere to $\frac{\partial}{\partial t}$, in terms of a Kähler form and complex local coordinates, Ω being the conformal factor. Functions H and Ω depend only on complex coordinates z and $\bar{z}$. Far from the finite-energy matter sources, it is well-known that the metric must approach a cone with deficit angle δ, whose explicit value will be derived below. This behaviour can be expressed in terms of the following asymptotic conditions for functions H and Ω,

$$H \to 1 \quad , \quad \Omega \to |z|^{-\delta/\pi} \ . \tag{12}$$

With this metric, the only non-vanishing components of the Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$, are

$$G_{00} = H^2 K - H \vec{\nabla}^2 H \ , \tag{13}$$

$$G_{ij} = -2H^{-1} \nabla_i \nabla_j H + k_{ij} H^{-2} (\vec{\nabla} H)^2 + 2 k_{ij} H^{-1} \vec{\nabla}^2 H \ , \tag{14}$$

where K is the Gauss curvature of the two-dimensional metric k_{ij} that spans Π, while ∇_i is the covariant derivative with respect to the planar metric. The integral of the Gauss curvature over the surface Π can be evaluated using the Gauss-Bonnet theorem:

$$\int_\Pi dz d\bar{z} \Omega^2 K = \delta - 4\pi g \ , \tag{15}$$

where δ is the deficit angle and g is the number of handles of the two-dimensional manifold Π. We will be concerned with the simplest case where the topology of Π is that of a two-disk, $g = 0$. The 00-component of the Einstein equations $G_{\mu\nu} = \kappa^2 T_{\mu\nu}^{\ mat}$, can be the used to obtain an explicit equation for the deficit angle,

$$\delta = \int_\Pi dz d\bar{z} \Omega^2 \left[\left(\vec{\nabla} \ln H \right)^2 + \frac{1}{M_{pl}} T^{0mat}_{\ 0} \right] \geq \frac{M}{M_{pl}} \ , \tag{16}$$

which results to be bounded by the total mass of the field configuration,

$$M = \frac{1}{2} \int dz d\bar{z} \left(\frac{\Omega}{H} \right)^2 \left[F_{z\bar{z}} F^{z\bar{z}} + (D_i \Phi)^\dagger (D^i \Phi) + \frac{e^2}{4} (\Phi^\dagger \Phi - v^2)^2 \right] . \tag{17}$$

The bound is saturated provided

$$\nabla_i \ln H = 0 \ , \tag{18}$$

holds locally, this implying that H is a constant which, after conditions (12), is taken to be one, $H = 1$.

As we have previously mentioned, the existence of a spinor that is asymptotically supercovariantly constant is related to its properties under parallel transport at infinity. Consider, for example, in pure supergravity, a spinor η with definite 'chirality', $\gamma^0\eta_\pm = \pm\eta_\pm$. Then, the parallel transport around a closed curve Γ of large radius R, surrounding all the static matter sources, is given by the following path-ordered integration:

$$\begin{aligned} \eta_\pm(R,2\pi) &= \mathcal{P}\exp\left(-\frac{i}{2}\oint_\Gamma \omega_\mu^a\gamma_a dx^\mu\right)\eta_\pm(R,0) \\ &= \exp\left[\pm i\pi\delta\right]\eta_\pm(R,0)\ . \end{aligned} \tag{19}$$

In view of (16), a non-trivial holonomy of geometric nature arises for masses below the Planck scale, this resulting in an ill-defined spinor.

In our model, however, the supercovariant derivative receives corrections from the fact that the gravitino is charged under the gauged $U(1)$ symmetry. Then, the previous argument slightly modifies to give:

$$\begin{aligned} \eta_\pm(R,2\pi) &= \mathcal{P}\exp\left(-\frac{i}{2}\oint_\Gamma \omega_\mu^a\gamma_a dx^\mu + \frac{i}{4M_{pl}}\oint_\Gamma (J_\mu + ev^2A_\mu)dx^\mu\right)\eta_\pm(R,0) \\ &= \mathcal{P}\exp\left(-\frac{i}{2}\oint_\Gamma \omega_\mu^a\gamma_a dx^\mu + i\frac{ev^2}{4M_{pl}}\oint_\Gamma A_\mu dx^\mu\right)\eta_\pm(R,0)\ , \end{aligned} \tag{20}$$

where we have imposed the constraint of finite energy on the Higgs field current. We inmediately see that the charged spinor η, acquires a Aharonov-Bohm phase provided some magnetic flux Φ_n exists across the surface delimited by Γ. Indeed, eq.(20) can be rewritten as

$$\eta_\pm(R,2\pi) = \exp\left[\pm i\pi\left(\delta \pm \frac{M_v^2}{4eM_{pl}}\Phi_n\right)\right]\eta_\pm(R,0)\ , \tag{21}$$

where $M_v^2 = e^2v^2$ is the 'photon' mass and Φ_n, as discussed above, is quantized:

$$\Phi_n = -\frac{2\pi n}{e}\ . \tag{22}$$

There are vortex configurations that solve the Bogomol'nyi equations of the system (first-order differential equations whose solutions also satisfy the more involved Euler-Lagrange ones), for which phases exactly cancel [11]. Then, asymptotical Killing spinors do exist over these 2+1-dimensional background solutions and their corresponding unbroken supercharges should be built. Let us analyse more carefully this issue in order to see whether it leads or not to an evasion of Witten's claim on the cosmological constant problem. We first construct the supercharge algebra to have a deeper understanding on its connection with the Bogomol'nyi bound of the system.

In analogy to what happens in four-dimensional theories [13, 14], the supercharge in three-dimensional supergravity is given by a surface integral

$$\mathcal{Q}[\epsilon] = 2M_{pl}^{1/2} \oint_{\partial\Sigma} \bar{\epsilon}\psi_\mu dx^\mu \ , \tag{23}$$

which is nothing but the circulation of the gravitino arround the oriented boundary $\partial\Sigma$ of a space-like surface Σ [7]. This quantity can not be used naively as a generator taking its Poisson brackets with a given field. One should first fix the whole gauge freedom such that the asymptotic value of the supersymmetry parameter determines its value everywhere [13, 15]. Alternatively, we can compute the algebra just by acting on the integrand of (23) with a supersymmetry transformation:

$$\{\bar{\mathcal{Q}}[\epsilon], \mathcal{Q}[\epsilon]\}| \equiv 2M_{pl}^{1/2} \oint_{\partial\Sigma} \bar{\epsilon}\delta_\epsilon\psi_\mu dx^\mu = 4M_{pl} \oint_{\partial\Sigma} \bar{\epsilon}\hat{\nabla}_\mu\epsilon dx^\mu \ . \tag{24}$$

This expression relates, as usual, the supercharge algebra evaluated in the purely bosonic sector with the circulation of a generalized Nester form $\bar{\epsilon}\hat{\nabla}\epsilon$. We now impose a chirality condition over the spinor $\gamma^0\epsilon_\pm = \pm\epsilon_\pm$, and choose appropriate asymptotic conditions on the fields (i.e. consistent with finite energy recquirements). In order to compute the integral, we further consider the contour of integration at large but finite radius R (to avoid infrared problems). Then, for static configurations, it is straightforward to obtain the result:

$$\{\bar{\mathcal{Q}}[\epsilon_\pm], \mathcal{Q}[\epsilon_\pm]\}| = 4\pi M_{pl} \left(\delta \pm \frac{M_v^2}{4eM_{pl}}\Phi_n\right) \epsilon^\dagger_{\pm\infty}\epsilon_{\pm\infty}\Theta^2(R) \ , \tag{25}$$

where $\Theta(R)$ gives the asymptoptic behaviour of the supersymmetry parameter

$$\epsilon_\pm \to \Theta(R)\epsilon_{\pm\infty} \ . \tag{26}$$

One can also compute the supercharge algebra in a different (longer but more illuminating) way. We can use Stokes' theorem to rewrite the r.h.s. of (24) as

$$\{\bar{\mathcal{Q}}[\epsilon_\pm],\mathcal{Q}[\epsilon_\pm]\}| = 2M_{pl}\int_\Sigma \epsilon^{\mu\nu\beta}\nabla_\beta(\bar{\epsilon}_\pm\hat{\nabla}_\mu\epsilon_\pm)d\Sigma_\nu \ . \tag{27}$$

Then, as the commutator of supercovariant derivatives is given by:

$$[\hat{\nabla}_\mu,\hat{\nabla}_\nu] = \frac{1}{2}R_{\mu\nu}{}^a\gamma_a + \frac{i}{4M_{pl}}(\partial_\mu J_\nu - \partial_\nu J_\mu + ev^2F_{\mu\nu}) \ , \tag{28}$$

after using Einstein equations and the explicit form of the supersymmetry transformation laws of the fermionic fields (9)-(10), we obtain:

$$\begin{aligned}\{\bar{\mathcal{Q}}[\epsilon_\pm],\mathcal{Q}[\epsilon_\pm]\}| &= 2M_{pl}\int_\Pi dzd\bar{z}\Omega^2\Big[\Big(\hat{\bar{\nabla}}\epsilon_\pm\Big)^\dagger\Big(\hat{\bar{\nabla}}\epsilon_\pm\Big) - \Big(\hat{\nabla}_i\epsilon_\pm\Big)^\dagger\Big(\hat{\nabla}^i\epsilon_\pm\Big)\\ &+ \frac{1}{2M_{pl}}\Big(\delta_{\epsilon_\pm}\Psi|^\dagger\delta_{\epsilon_\pm}\Psi| + \delta_{\epsilon_\pm}\lambda|^\dagger\delta_{\epsilon_\pm}\lambda|\Big)\Big] \ ,\end{aligned} \tag{29}$$

where we have specialized our spacelike integration surface Σ so that $d\Sigma_\mu = (d\Sigma_0,\vec{0})$. At this point, we note that after imposing a generalized Witten condition [16] on the spinorial parameter ϵ

$$\gamma^i\hat{\nabla}_i\epsilon_\pm = 0, \tag{30}$$

the asymptotic behaviour of ϵ can be determined and the r.h.s of eq.(29) results to be a sum of bilinear terms hence semi-positive definite[1]:

$$\{\bar{\mathcal{Q}}[\epsilon],\mathcal{Q}[\epsilon]\}| \geq 0 \ . \tag{31}$$

In view of (25), this inequality corresponds to a bound on the deficit angle

$$\delta \geq \mp\frac{M_v^2}{4eM_{pl}}\Phi_n \ , \tag{32}$$

which results to be saturated if and only if

$$\delta_{\epsilon_\pm}\Psi = \delta_{\epsilon_\pm}\lambda = 0 \ , \tag{33}$$

[1] It is natural to impose such a condition in order to consistently eliminate gauge degrees of freedom. In fact, unphysical excitations are eliminated in the transverse gauge $\gamma^i\psi_i = 0$, and this gauge fixing is respected by supersymmetry provided ϵ satisfies the generalized Witten condition (30).

$$\delta_{\epsilon_\pm}\psi_\mu = 0 \ . \tag{34}$$

We recognize in the first couple of equations the Bogomol'nyi equations for matter fields

$$\mathcal{F} = \frac{e}{2}(\Phi^\dagger\Phi - v^2) = 0 \quad , \quad D_z\Phi = 0 \ , \tag{35}$$

for the upper sign, and

$$\mathcal{F} = -\frac{e}{2}(\Phi^\dagger\Phi - v^2) = 0 \quad , \quad D_{\bar{z}}\Phi = 0 \ . \tag{36}$$

for the lower one. Here $\mathcal{F} = \epsilon^{z\bar{z}}F_{z\bar{z}}$, where $\epsilon^{z\bar{z}}$ is the covariant antisymmetric tensor. These equations were originally found in Ref.[11] for the bosonic system in a non-supersymmetric context. Let us remark that we have obtained them just by asking our configuration to have an unbroken supersymmetry or, in other words, to saturate the lower bound that results from the supercharge algebra. Concerning equation (34), its solvability can be studied from the integrability conditions

$$[\hat{\nabla}_\mu, \hat{\nabla}_\nu]\epsilon_\pm = 0 \ , \tag{37}$$

which happen to be equivalent to the Einstein equations of the purely bosonic system, once Bogomol'nyi equations (35) or (36), according to the chirality of ϵ, have been imposed. This leaves us with an exact Killing spinor preserved by this $2+1$-dimensional solitonic background. Even though a naïve analysis of the vacuum manifold of this system would lead us to conclude that it has a trivial topology, we have shown that the no-go scenario for the existence of Killing spinors in an asymptotically conical space-time is indeed overcome. This further extends the class of $2+1$-dimensional systems studied in [8], that admits Killing spinors over massive configurations.

Let us end this letter by considering the connection between the unbroken supersymmetry found above, and the cosmological constant problem in the context of supergravity theories. In order to attempt the construction of the entire massive supermultiplet associated to a given semi-local cosmic string (that saturates the Bogomol'nyi bound), we must apply the broken supersymmetry generator over our purely bosonic configuration. Let us assume that the Killing spinor has chirality ϵ_+ and equations (35) hold, then:

$$\delta_{\epsilon_-}\lambda| = \frac{ie}{2}(\Phi^\dagger\Phi - v^2)\epsilon_- \neq 0 \quad , \quad \delta_{\epsilon_-}\Psi| = \frac{1}{2}\gamma^{\bar{z}}D_{\bar{z}}\Phi\epsilon_- \neq 0 \ , \tag{38}$$

$$\delta_{\epsilon_-}\psi_\mu| = 2M_{pl}^{1/2}\hat{\nabla}_\mu\epsilon_- \neq 0 \ , \qquad (39)$$

are nothing but the Nambu-Goldstone fermionic zero mode corresponding to the unbroken supersymmetry. We note that the asymptotic behaviour of ϵ_- given in (26) leads, for masses below the Planck scale, to an infrared divergent contribution coming from eq.(39), that renders the zero mode non-normalizable. As a consequence, the zero mode must not be used to construct the physical Hilbert space of the theory. That is, though there seem to exist unbroken supersymmetries over certain massive configurations, they cannot be realized on the physical spectrum. Hence, in this model, the vanishing of the cosmological constant implied by the supersymmetries of the vacuum, does not compell bosons and fermions to be degenerate. It would be very interesting to generalize this mechanism to $3+1$-dimensional systems. We hope to report on this issue in a forthcoming work.

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