Bogomol'nyi-Prasad-Sommerfield analysis of gauge-field–Higgs models in nonanticommutative superspace

L. G. Aldrovandi, D. H. Correa, F. A. Schaposnik,* and G. A. Silva†

Departamento de Física, Universidad Nacional de La Plata, C.C. 67, (1900) La Plata, Argentina

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We extend the study of Bogomol'nyi-Prasad-Sommerfield (BPS) equations in $\mathcal{N} = 1/2$ super Yang-Mills theory to the case of models with gauge symmetry breaking. We first consider an Abelian gauge-Higgs supersymmetric Lagrangian in $d = 4$ dimensional Euclidean space obtained by deforming $\mathcal{N} = 1$ superspace. The supermultiplets include chiral and vector superfields and its bosonic content coincides with that of the Abelian-Higgs model where vortex solutions to the BPS equation are known to exist in the undeformed case. We also consider the $d = 3$ dimensional reduction of a non-Abelian $d = 4$ deformed model and study its deformed BPS equations, showing the existence of new monopole solutions which depend on the deformation parameter.

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I. INTRODUCTION

Nonanticommutative (NAC) theories recently attracted much attention because of their relation with superstring effective actions in backgrounds with constant graviphoton field strength [1–4]. They can be constructed, within the superfield formulation of supersymmetric (SUSY) theories, by introducing different deformations in the odd superspace variables algebra [5–10]. As in ordinary noncommutative space, one can introduce a Moyal star product to multiply superfields entering in the construction of NAC Lagrangians. Depending on whether one chooses the supercovariant derivatives D_α [7] or the supersymmetric generators Q_α [3] to define such star product, one obtains a supersymmetric (but chirality nonpreserving) theory or a partially supersymmetric (but chirality preserving) one. Following this last approach, Seiberg [3] studied $\mathcal{N} = 1$ superspace and constructed a super Yang-Mills Lagrangian in $d = 4$ Euclidean space which differs from the undeformed one in a polynomial in the deformation parameter with terms containing fermion bilinear products. The resulting deformation reduces the supersymmetry of the action from $\mathcal{N} = 1$ to $\mathcal{N} = 1/2$.

In order to study nonperturbative aspects of Seiberg's $\mathcal{N} = 1/2$ super Yang-Mills theory, instanton solutions were constructed in [11–14]. As stressed in [3], if one restricts the analysis to the purely bosonic sector (putting fermions to zero) self-duality and anti-self-duality equations are not modified. One can study however how the bosonic equations get modified when fermions are turned on. One possibility is to arrange the action functional into perfect squares. Now, since the deformed action is in general complex, the first order equations resulting from the vanishing of the squares should be understood as corresponding to an enhancement of the symmetry instead of leading to a minimum of the action. Interestingly enough, one finds that only the anti-self-duality equations are modified in the deformed super Yang-Mills theory [11–13]. Following a similar approach, some soliton solutions in $d = 2$ NAC theories were discussed [15–18].

As it is well known, vortex and monopole Bogomol'nyi-Prasad-Sommerfield (BPS) equations can be obtained by studying the supersymmetric extension of Abelian and non-Abelian gauge theories coupled to Higgs scalars [19–23]. It is the purpose of the present work to extend this analysis to the case of NAC theories. We start in section II by discussing deformed superspace and then consider a gauge-Higgs supersymmetric Lagrangian in $d = 4$ dimensional Euclidean space obtained by deforming $\mathcal{N} = 1$ superspace. The supermultiplets include chiral and vector superfields (containing a complex scalar and a U(1) gauge field, respectively) so that the bosonic content coincides with that of the Abelian-Higgs model. We show in section III that although new terms arise due to the deformation, the resulting equations of motion and supersymmetry transformations show that no consistent first order Bogomol'nyi equations arise except when fermions are turned off (thus eliminating the deformation effects). That is, in contrast with what happens in the instanton case, one cannot find deformed vortex configurations solutions. In order to make a similar analysis for monopoles we discuss in section IV the $d = 3$ dimensional reduction of a non-Abelian $d = 4$ deformed model. In this case one gets deformed BPS equations and new monopole solutions which depend on the deformation parameter. We present a discussion of our results in section V. We give in an Appendix some conventions adopted in our calculations.

II. DEFORMED SUPERSPACE

We shall consider the deformation of four-dimensional Euclidean $\mathcal{N} = 1$ superspace parametrized by superspace bosonic coordinates x^μ and chiral and antichiral fermionic coordinates θ^α, $\bar\theta^{\dot\alpha}$ as introduced in [3]

*F. A. S. is associated with CICBA.
†G. A. S. is associated with CONICET.

$$\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta}, \qquad \{\bar\theta^{\dot\alpha}, \bar\theta^{\dot\beta}\} = 0, \qquad \{\theta^\alpha, \bar\theta^{\dot\beta}\} = 0. \tag{1}$$

Here $C^{\alpha\beta}$ are constant elements of a symmetric matrix. Defining chiral and antichiral coordinates according to

$$y^\mu = x^\mu + i\theta\sigma^\mu\bar\theta, \tag{2}$$

$$\bar y^\mu = y^\mu - 2i\theta\sigma^\mu\bar\theta, \tag{3}$$

one imposes [3]

$$[y^\mu, y^\nu] = [y^\mu, \theta^\alpha] = [y^\mu, \bar\theta^{\dot\alpha}] = 0 \tag{4}$$

and obtains as a consequence of (1)–(4)

$$[\bar y^\mu, \bar y^\nu] = 4\bar\theta\,\bar\theta\,\mathbf{C}^{\mu\nu}, \tag{5}$$

where $\mathbf{C}^{\mu\nu} = C^{\alpha\beta}(\sigma^{\mu\nu})_{\alpha\beta}$ is antisymmetric and anti-self-dual (see the Appendix for conventions on gamma matrices and spinors).

The nonanticommutative field theory in such a deformed superspace can be defined in terms of superfields that are multiplied according to the following Moyal product [3]

$$\Phi(y,\theta,\bar\theta) * \Psi(y,\theta,\bar\theta) = \Phi(y,\theta,\bar\theta)\exp\left(-\frac{C^{\alpha\beta}}{2}\frac{\overleftarrow{\partial}}{\partial\theta^\alpha}\frac{\overrightarrow{\partial}}{\partial\theta^\beta}\right) \times \Psi(y,\theta,\bar\theta). \tag{6}$$

Supercharges and covariant derivatives in chiral coordinates take the form

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha}, \qquad \bar Q_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + 2i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\frac{\partial}{\partial y^\mu}, \tag{7}$$

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + 2i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\frac{\partial}{\partial y^\mu}, \qquad \bar D_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}}. \tag{8}$$

The $D - D$ algebra is not modified by the deformation (1) as happens for the $Q - D$ and $\bar Q - D$ algebra. Concerning the supercharge algebra, it is modified according to

$$\{\bar Q_{\dot\alpha}, Q_\alpha\} = 2i\sigma^\mu_{\alpha\dot\alpha}\frac{\partial}{\partial y^\mu}, \tag{9}$$

$$\{Q_\alpha, Q_\beta\} = 0, \tag{10}$$

$$\{\bar Q_{\dot\alpha}, \bar Q_{\dot\beta}\} = -4C^{\alpha\beta}\sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta}\frac{\partial^2}{\partial y^\mu\partial y^\nu}. \tag{11}$$

Then, only the subalgebra generated by Q_α is still preserved and this defines the chiral $\mathcal{N} = 1/2$ supersymmetry algebra [3].

A chiral superfield Φ satisfying $\bar D_{\dot\alpha}\Phi = 0$ can be, as usual, written in the form

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y). \tag{12}$$

As a consequence of (5) an ordering must be chosen for the antichiral field $\bar\Phi(\bar y, \bar\theta)$, a natural one is given by expressing it in terms of the chiral variable y^μ, it then takes the form

$$\begin{aligned}\bar\Phi(y - 2i\theta\sigma\bar\theta, \bar\theta) = {}& \bar\phi(y) + \sqrt{2}\bar\theta\,\bar\psi(y) - 2i\theta\sigma^\mu\bar\theta\partial_\mu\bar\phi(y) \\ &+ \bar\theta\,\bar\theta[\bar F(y) + i\sqrt{2}\theta\sigma^\mu\partial_\mu\bar\psi(y) \\ &+ \theta\theta\partial^\mu\partial_\mu\bar\phi].\end{aligned} \tag{13}$$

Now, let us consider a vector superfield V containing the gauge field for a group G. We take t^a as basis of the Lie algebra satisfying $[t^a, t^b] = if^{abc}t^c$ and $\mathrm{tr}(t^a t^b) = \frac{1}{2}\delta^{ab}$. A gauge transformation acts as

$$\begin{aligned}\exp(-2gV) \to{}& \exp(-2gV') \\ &= \exp(ig\bar\Lambda) * \exp(-2gV) * \exp(-ig\Lambda),\end{aligned} \tag{14}$$

where Λ and $\bar\Lambda$ are chiral and antichiral fields in the Lie algebra of G. In all the expressions above exponentials are defined through their $*$-product expansion,

$$\exp(i\Omega) = 1 + i\Omega + \frac{i^2}{2}\Omega * \Omega + \ldots \tag{15}$$

For the chiral and antichiral superfield strengths, the standard expressions hold,

$$\begin{aligned}W_\alpha &= \frac{1}{8g}\bar D * \bar D * \exp(2gV) * D_\alpha * \exp(-2gV), \\ \bar W_{\dot\alpha} &= -\frac{1}{8g}D * D * \exp(-2gV) * \bar D_{\dot\alpha} * \exp(2gV),\end{aligned} \tag{16}$$

transforming under gauge rotations according to

$$\begin{aligned}W_\alpha &\to \exp(ig\Lambda) * W_\alpha * \exp(-ig\Lambda), \\ \bar W_{\dot\alpha} &\to \exp(ig\bar\Lambda) * \bar W_{\dot\alpha} * \exp(-ig\bar\Lambda).\end{aligned} \tag{17}$$

Infinitesimally we have

$$\delta W = ig[\Lambda, W]_*, \qquad \delta\bar W = ig[\bar\Lambda, \bar W]_*. \tag{18}$$

Since the commutator involves matrix and Moyal products, as in standard noncommutative gauge theories one should consider groups closing their Lie algebra generators under anticommutation.

We want to write the vector superfield in the Wess-Zumino gauge. As in ordinary superspace this is achieved by exploiting the gauge freedom (14) to set some of the components of V to zero. In the generalization to nonanticommutative theory the vector superfield V in the Wess-Zumino gauge takes the form [3]

$$\begin{aligned}V(y,\theta.\bar\theta) = {}& -\theta\sigma^\mu\bar\theta A_\mu(y) - i\bar\theta\,\bar\theta\,\theta^\alpha[\lambda_\alpha(y) \\ &- \frac{g}{2}\varepsilon_{\alpha\beta}C^{\beta\gamma}\sigma^\mu_{\gamma\dot\gamma}\{\bar\lambda^{\dot\gamma}(y), A_\mu(y)\}] + i\theta\theta\bar\theta\,\bar\lambda(y) \\ &+ \frac{1}{2}\theta\theta\bar\theta\,\bar\theta[D(y) - i\partial_\mu A^\mu(y)].\end{aligned} \tag{19}$$

This leads to

$$V_*^2 \equiv -\frac{1}{2}\bar\theta\,\bar\theta\Big(\theta\theta A_\mu A^\mu + \mathbf{C}^{\mu\nu}A_\mu A_\nu - i\theta_\alpha C^{\alpha\beta}\sigma^\mu_{\beta\dot\beta}[A_\mu, \bar\lambda^{\dot\beta}] + \frac{1}{4}|\mathbf{C}|^2\bar\lambda\,\bar\lambda\Big), \qquad V_*^3 = 0, \tag{20}$$

where $|\mathbf{C}|^2 = \mathbf{C}^{\mu\nu}\mathbf{C}_{\mu\nu}$. One can still perform gauge transformations preserving Wess-Zumino gauge (19) through

$$\Lambda = \varphi(y),$$
$$\bar\Lambda = \varphi(y) - 2i\theta\sigma^\mu\bar\theta\partial_\mu\varphi(y) + \theta\theta\bar\theta\,\bar\theta\,\partial^\mu\partial_\mu\varphi(y) - ig\bar\theta\,\bar\theta\,\mathbf{C}^{\mu\nu}\{\partial_\mu\varphi(y), A_\nu(y)\}. \tag{21}$$

In components this gauge transformation reads

$$\delta A_\mu = \mathcal{D}_\mu\varphi \equiv \partial_\mu\varphi - ig[A_\mu,\varphi],$$
$$\delta\lambda_\alpha = -ig[\lambda_\alpha,\varphi], \qquad \delta\bar\lambda_{\dot\alpha} = -ig[\bar\lambda_{\dot\alpha},\varphi], \tag{22}$$
$$\delta D = -ig[D,\varphi].$$

Chiral superfields charged under the gauge group transform according to

$$\Phi \to \exp(ig\Lambda) * \Phi, \qquad \bar\Phi \to \bar\Phi * \exp(-ig\bar\Lambda). \tag{23}$$

As in the case of the vector superfield in Eq. (19), a C-dependent term is needed in the parametrization of antichiral matter superfields in order for the field components to have the ordinary gauge transformation [10]

$$\bar\Phi(\bar y, \bar\theta) = \bar\phi(\bar y) + \sqrt{2}\bar\theta\,\bar\psi(\bar y) + \bar\theta\,\bar\theta\{\bar F(\bar y) + 2ig\mathbf{C}^{\mu\nu}\partial_\mu[\bar\phi(\bar y)A_\nu(\bar y)] + g^2\mathbf{C}^{\mu\nu}\bar\phi(\bar y)A_\mu(\bar y)A_\nu(\bar y)\}. \tag{24}$$

Then, written in components, infinitesimal gauge transformations read

$$\delta\phi = ig\varphi\phi, \qquad \delta\bar\phi = -ig\bar\phi\varphi, \qquad \delta\psi = ig\varphi\psi,$$
$$\delta\bar\psi = -ig\bar\psi\varphi, \qquad \delta F = ig\varphi F, \qquad \delta\bar F = -ig\bar F\varphi. \tag{25}$$

III. SUPERSYMMETRIC MAXWELL-HIGGS MODEL IN $d = 4$ AND DEFORMED VORTICES

The $d = 4$ deformed supersymmetric Maxwell-Higgs model is constructed with the multiplets discussed in the previous section as

$$\mathcal{L} = \int d^2\theta d^2\bar\theta[\bar\Phi * \exp(-2gV) * \Phi + 2gv_0^2V] + \frac{1}{4}\Big(\int d^2\theta W * W + \int d^2\bar\theta\,\bar W * \bar W\Big), \tag{26}$$

where all superfields are multiplied using the Moyal product (6). A Fayet-Iliopoulos term has been included in order to achieve spontaneous gauge symmetry breaking. In components, the Lagrangian reads

$$\mathcal{L} = \mathcal{L}_b + \mathcal{L}_f, \tag{27}$$

where

$$\mathcal{L}_b = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \overline{D_\mu\phi}D^\mu\phi - gD(\bar\phi\phi - v_0^2) + \frac{1}{2}D^2 + \bar F F - ig\mathbf{C}^{\mu\nu}\bar\phi F_{\mu\nu}F,$$
$$\mathcal{L}_f = -i\bar\lambda\bar\sigma^\mu\partial_\mu\lambda - i\bar\psi\bar\sigma^\mu D_\mu\psi - i\sqrt{2}g(\bar\phi\lambda\psi - \bar\psi\,\bar\lambda\,\phi) + ig\mathbf{C}^{\mu\nu}F_{\mu\nu}\bar\lambda\,\bar\lambda + \sqrt{2}gC^{\alpha\beta}\sigma^\mu_{\alpha\dot\alpha}\overline{D_\mu\phi}\bar\lambda^{\dot\alpha}\psi_\beta - \frac{g^2}{4}|\mathbf{C}|^2\bar\phi\,\bar\lambda\,\bar\lambda\,F. \tag{28}$$

Here

$$D_\mu = \partial_\mu - igA_\mu, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{29}$$

The transformation laws associated with the $\mathcal{N} = 1/2$ surviving supersymmetry read

$$\delta\phi = \sqrt{2}\xi\psi, \qquad \delta\bar\phi = 0, \qquad \delta\psi_\alpha = \sqrt{2}\xi_\alpha F,$$
$$\delta\bar\psi_{\dot\alpha} = -i\sqrt{2}\overline{D_\mu\phi}(\xi\sigma^\mu)_{\dot\alpha}, \qquad \delta F = 0,$$
$$\delta\bar F = -i\sqrt{2}\overline{D_\mu\psi}\bar\sigma^\mu\xi + 2ig\bar\phi\xi\lambda - 2g\mathbf{C}^{\mu\nu}[\partial_\mu(\bar\phi\xi\sigma_\nu\bar\lambda) + ig(\bar\phi\xi\sigma_\nu\bar\lambda)A_\mu], \tag{30}$$
$$\delta A_\mu = -i\bar\lambda\bar\sigma_\mu\xi \to \delta F_{\mu\nu} = -i(\partial_\mu\bar\lambda\bar\sigma_\nu - \partial_\nu\bar\lambda\bar\sigma_\mu)\xi,$$
$$\delta\lambda_\alpha = i\xi_\alpha D + (\sigma^{\mu\nu}\xi)_\alpha(F_{\mu\nu} - ig\mathbf{C}_{\mu\nu}\bar\lambda\,\bar\lambda),$$
$$\delta\bar\lambda_{\dot\alpha} = 0, \qquad \delta D = -\xi\sigma^\mu\partial_\mu\bar\lambda.$$

The second order equations of motion associated to the Lagrangian (28) are

$$\partial_\mu F^{\mu\nu} = ig(\bar\phi D^\nu\phi - \phi\overline{D^\nu\phi}) + g\bar\psi\bar\sigma^\nu\psi + 2ig\mathbf{C}^{\mu\nu}\partial_\mu(\bar\lambda\,\bar\lambda - \bar\phi F) - ig^2\sqrt{2}C^{\alpha\beta}\sigma^\nu_{\alpha\dot\alpha}\bar\phi\bar\lambda^{\dot\alpha}\psi_\beta, \tag{31}$$

$$D^\mu D_\mu\phi = g\phi D + ig\sqrt{2}\lambda\psi + ig\mathbf{C}^{\mu\nu}F_{\mu\nu}F + \sqrt{2}gC^{\alpha\beta}\sigma^\mu_{\alpha\dot\alpha}D_\mu(\bar\lambda^{\dot\alpha}\psi_\beta) + g^2\frac{|\mathbf{C}|^2}{4}\bar\lambda\,\bar\lambda\,F, \tag{32}$$

$$\overline{D^\mu}\,\overline{D_\mu\phi} = g\bar\phi D - ig\sqrt{2}\bar\lambda\,\bar\psi, \tag{33}$$

$$F = 0, \qquad \bar F = ig\bar\phi\mathbf{C}^{\mu\nu}F_{\mu\nu} + g^2\frac{|\mathbf{C}|^2}{4}\bar\phi\,\bar\lambda\,\bar\lambda, \tag{34}$$

$$D = g(\bar\phi\phi - v_0^2), \tag{35}$$

$$(\sigma^\mu\partial_\mu\bar\lambda)_\alpha = -\sqrt{2}g\bar\phi\psi_\alpha, \tag{36}$$

$$(\bar{\sigma}^{\mu}\partial_{\mu}\lambda)_{\dot{\alpha}} = \sqrt{2}g\phi\bar{\psi}_{\dot{\alpha}} + 2g\mathbf{C}^{\mu\nu}F_{\mu\nu}\bar{\lambda}_{\dot{\alpha}} + i\sqrt{2}g\overline{D_{\mu}\phi}\psi_{\alpha}C^{\alpha\beta}\sigma^{\mu}_{\beta\dot{\alpha}} + ig^2\frac{|\mathbf{C}|^2}{2}F\bar{\phi}\bar{\lambda}_{\dot{\alpha}}, \tag{37}$$

$$(\bar{\sigma}^{\mu}D_{\mu}\psi)_{\dot{\alpha}} = \sqrt{2}g\phi\bar{\lambda}_{\dot{\alpha}}, \tag{38}$$

$$(\sigma^{\mu}\overline{D_{\mu}\psi})_{\beta} = -\sqrt{2}g\bar{\phi}\lambda_{\beta} - i\sqrt{2}g\overline{D_{\mu}\phi}\bar{\lambda}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}C_{\alpha\beta}. \tag{39}$$

In connection with gauge symmetry breaking it is interesting to look for constant solutions to Eqs. (31)–(39) to see whether the usual Higgs vacuum ($\phi = v_0$, $A_{\mu} = \partial_{\mu}\Lambda$) is modified by the deformation. In particular, one could think that the presence of new C-dependent terms could lead to gauge symmetry breaking even when $v_0^2 = 0$. This possibility is suggested by the supersymmetry variation of λ which exhibits a term proportional to $C\bar{\lambda}\,\bar{\lambda}$, which could play the role that v_0^2 does in the normal case. Now, for constant fields, the only equation involving the deformation parameter C is (31)

$$\bar{\psi}\bar{\sigma}^{\nu}\psi = ig\sqrt{2}C^{\alpha\beta}\sigma^{\nu}_{\alpha\dot{\alpha}}\bar{\phi}\bar{\lambda}^{\dot{\alpha}}\psi_{\beta}. \tag{40}$$

In order to have a nontrivial C contribution we need $\bar{\phi}$, ψ, and $\bar{\lambda}$ to be nonvanishing constants. However this is not possible in view of Eq. (36). We then conclude that there is no nontrivial symmetry breaking mechanism apart from that originated by the standard Fayet-Iliopoulos term.

In the $d = 4$ super Yang-Mills theory case, instanton configurations were constructed by solving a deformed version of the first order self-duality equations [11–13]. The deformation was originated by the presence of fermionic zero modes. One could expect that in the present case, deformed Nielsen-Olesen vortex configurations could be obtained by solving some deformed first order BPS equations. To this end, let us restrict fields, from here on, to the x^1, x^2 plane and make $A_3 = A_4 = 0$. Moreover, we shall consider for simplicity that the only nonvanishing $\mathbf{C}^{\mu\nu}$ components are $\mathbf{C}_{12} = -\mathbf{C}_{34}$.

In the undeformed case, BPS equations can be obtained from the vanishing of the supersymmetry transformations for fermions, once the auxiliary fields are put on shell. Nontrivial solutions to these equations are invariant under $1/2$ of the original supersymmetries. Let us then analyze the $\mathcal{N} = 1/2$ surviving supersymmetry variations (30). There are two possibilities for making the supersymmetry variations of the fermionic and the auxiliary fields vanish: either $\xi_1 = 0$ and the following first order equations hold ("anti-self-dual case")

$$F_{12} = g(\bar{\phi}\phi - v_0^2) - i\mathbf{C}_{12}\bar{\lambda}\,\bar{\lambda}, \tag{41}$$

$$\overline{D_1\phi} + i\overline{D_2\phi} = 0, \tag{42}$$

$$\sqrt{2}(\bar{D}_1 - i\bar{D}_2)\bar{\psi}_{\dot{1}} + \bar{\phi}\lambda^2 - \mathbf{C}_{12}\bar{\lambda}_{\dot{1}}(\overline{D_1\phi} - i\overline{D_2\phi}) = 0, \tag{43}$$

$$(\partial_1 - i\partial_2)\bar{\lambda}^{\dot{2}} = 0, \qquad F = 0, \tag{44}$$

or $\xi_2 = 0$ and the first order equations take the form ("self-dual case")

$$F_{12} = -g(\bar{\phi}\phi - v_0^2) - i\mathbf{C}_{12}\bar{\lambda}\,\bar{\lambda}, \tag{45}$$

$$\overline{D_1\phi} - i\overline{D_2\phi} = 0, \tag{46}$$

$$\sqrt{2}(\bar{D}_1 + i\bar{D}_2)\bar{\psi}_{\dot{2}} + \bar{\phi}\lambda^1 + \mathbf{C}_{12}\bar{\lambda}_{\dot{2}}(\overline{D_1\phi} + i\overline{D_2\phi}) = 0, \tag{47}$$

$$(\partial_1 + i\partial_2)\bar{\lambda}^{\dot{1}} = 0, \qquad F = 0. \tag{48}$$

At this point, an important difference with respect to the Yang-Mills deformed case should be stressed. In the latter, for anti-self-dual configurations, fermions are invariant under the whole $\mathcal{N} = 1/2$ surviving symmetry while for self-dual configurations they are not. In the present case, according to (41)–(48) both for self-dual and anti-self-dual configurations fermions would be invariant under $1/2$ of the $\mathcal{N} = 1/2$ supersymmetry which survived the deformation.

Let us discuss, for definiteness, the self-dual case (Eqs. (41)–(44), the anti-self-dual one goes the same). Compatibility of Eq. (44) for $\bar{\lambda}^{\dot{2}}$ with equation of motion (36) implies that $\psi_1 = 0$. But this in turn implies, because of Eq. (38), that $\bar{\lambda}^{\dot{2}} = 0$, so that finally $\bar{\lambda}\,\bar{\lambda} = 0$: any effect from deformation is finally washed out.

In brief, on the one hand one necessarily has to keep $\bar{\lambda}\,\bar{\lambda} \neq 0$ in order to discover new features in the deformed model. On the other hand, the deformed first order BPS equations obtained from the vanishing of supersymmetry transformations are not compatible with the equations of motion, except if some fermionic fields vanish turning the deformed BPS equations into the undeformed (ordinary) ones.

The previous results can be also understood by noting that in fact the deformed Lagrangian cannot be arranged as a sum of perfect squares whose vanishing lead to deformed first order Eqs. (41)–(44), as one can do in the undeformed case. Indeed, one cannot reproduce Lagrangian (28) from, among others, a square term of the form

$$(F_{12} - D - i\mathbf{C}_{12}\bar{\lambda}\,\bar{\lambda})^2, \tag{49}$$

since a term of the form $iD\mathbf{C}_{12}\bar{\lambda}\,\bar{\lambda}$ is lacking in Eq. (28). This again should be contrasted with the case of deformed Yang-Mills theory, where Lagrangian can be written as squares of C-deformed self-duality equations.

One can consider the possibility of finding C-dependent solutions by directly analyzing the equations of motion (restricted to the x^1, x^2 plane and with $A_3 = A_4 = 0$).

However, the set of coupled nonlinear equations lead to very complicated constraints. For example, Maxwell Eqs. (31) for $\nu = 3, 4$ require

$$\bar\psi\bar\sigma^\nu\psi = ig\sqrt{2}C^{\alpha\beta}\sigma^\nu_{\alpha\dot\alpha}\bar\phi\bar\lambda^{\dot\alpha}\psi_\beta, \quad \text{for } \nu = 3, 4 \tag{50}$$

and for nonvanishing ψ_α this leads to the constraints

$$\bar\psi^{\dot 1} = +\sqrt{2}igC^{12}\bar\phi\bar\lambda^{\dot 1}, \qquad \bar\psi^{\dot 2} = -\sqrt{2}igC^{12}\bar\phi\bar\lambda^{\dot 2}. \tag{51}$$

This in turn implies $\bar\psi\,\bar\lambda = 0$. We were not able to establish the compatibility of this result with the equation of motion for $\bar\psi$ and although a nontrivial C-dependent vortex solution cannot be *a priori* excluded, it seems extremely difficult to fulfill all the resulting constraints.

IV. SUPERSYMMETRIC U(2) YANG-MILLS-HIGGS MODEL IN $d = 3$ AND DEFORMED MONOPOLES

In this section we shall consider a deformed $d = 3$ supersymmetric U(2) gauge theory coupled to scalars in order to analyze possible modifications, induced by the deformation, on the BPS (first order) monopole equations. To this end, we start from a deformed supersymmetric $d = 4$ Yang-Mills theory and proceed to a dimensional reduction in which the A_4 component of the gauge field is identified with a Higgs field. The $d = 4$ Lagrangian in term of superfields reads

$$\mathcal{L} = \frac{1}{2}\mathrm{tr}\left(\int d^2\theta W * W + \int d^2\bar\theta\,\bar W * \bar W\right). \tag{52}$$

To write the Lagragian in components, we use Dirac spinors and a generic Γ matrices representation,

$$\mathcal{L} = \mathrm{tr}\Big[-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - i\Lambda^C\Gamma^\mu\mathcal{D}_\mu\Lambda + 2ig\mathbf{C}^{\mu\nu}F_{\mu\nu}\Lambda^C P_-\Lambda + g^2|\mathbf{C}|^2(\Lambda^C P_-\Lambda)^2 + D^2\Big], \tag{53}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu], \qquad \mathcal{D}_\mu\Lambda = \partial_\mu\Lambda - ig[A_\mu, \Lambda]. \tag{54}$$

Here we write $A_\mu = A^a_\mu t^a$ with t^a the hermitic generators normalized according to $\mathrm{tr}t^a t^b = \frac{1}{2}\delta^{ab}$. For the present U(2) case $t^a = \sigma^a/2$ ($a = 1, 2, 3$) and $t^4 = I/2$.

The equations of motion derived from Lagrangian (53) are

$$\mathcal{D}_\mu F^{\mu\nu} = 2ig\mathbf{C}^{\mu\nu}\mathcal{D}_\mu(\Lambda^C P_-\Lambda) + \frac{g}{2}\{\Lambda^C_\rho, (\Gamma^\nu\Lambda)^\rho\},$$
$$\Gamma^\mu D_\mu\Lambda = g\mathbf{C}_{\mu\nu}\{F^{\mu\nu} - ig\mathbf{C}^{\mu\nu}\Lambda^C P_-\Lambda, P_-\Lambda\}, \tag{55}$$
$$D = 0.$$

In the dimensional reduction, a vector field in $d = 4$ becomes a vector field and a scalar field in $d = 3$

$$A_\mu \to A_i, \phi, \tag{56}$$

where $\phi = \phi^a t^a$ ($a = 1, 2, 3, 4$) will play the role of a Higgs field in the adjoint in $d = 3$. Concerning fermions, the $d = 4$ Dirac spinor Λ reduces to two $d = 3$ Dirac spinors Λ_1 and Λ_2,

$$\Lambda = \begin{pmatrix}\Lambda^1\\ \Lambda^2\end{pmatrix} \to \Lambda^1, \Lambda^2. \tag{57}$$

For later convenience, we redefine $d = 3$ fermions in the form

$$\eta = \frac{1}{\sqrt{2}}(\Lambda^1 + i\Lambda^2), \qquad \chi = \frac{1}{\sqrt{2}}(\Lambda^1 - i\Lambda^2). \tag{58}$$

The dimensionally reduced $d = 3$ Lagrangian then reads

$$\mathcal{L} = \mathrm{tr}\Big[-\frac{1}{2}F_{ij}F^{ij} - D^i\phi D_i\phi - 2i\chi^C\gamma^i D_i\eta + 2g\chi^C(\phi, \eta) + D^2 + 2ig\mathbf{C}^{ij}(F_{ij} + \varepsilon_{ijk}D^k\phi)\chi^C\chi + 2g^2\mathbf{C}^{ij}\mathbf{C}_{ij}(\chi^C\chi)^2\Big]. \tag{59}$$

Here the Majorana conjugates should be computed by using C_3. The equations of motion for the bosonic fields read

$$D_i(F^{ij} + ig\mathbf{C}^{ij}\chi^C\chi) = ig[D^j\phi + ig\varepsilon^{jkl}\mathbf{C}_{kl}\chi^C\chi, \phi] - \frac{g}{2}\eta^C\gamma^i\chi, \tag{60}$$

$$D^iD_i\phi = ig\varepsilon^{ijk}\mathbf{C}_{jk}D_i(\chi^C\chi) + \frac{g}{2}[\chi^C, \eta]. \tag{61}$$

Concerning fermions,

$$i\gamma^i D_i\chi - g[\phi, \chi] = 0, \tag{62}$$

$$i\gamma^i D_i\eta + g[\phi, \eta] = ig\mathbf{C}^{ij}\{\chi, F_{ij} - \varepsilon_{ijk}D^k\phi - 2ig\mathbf{C}_{ij}\chi^C\chi\}. \tag{63}$$

The $d = 3$ infinitesimal transformations associated with the supersymmetry read

$$\delta\eta = -\gamma^i\xi\Big[\frac{1}{2}\varepsilon_{ijk}(F^{jk} - 2ig\mathbf{C}^{jk}\chi^C\chi) + D_i\phi\Big] + iD\xi, \tag{64}$$

$$\delta\chi = 0, \tag{65}$$

$$\delta D = -\xi^C(\gamma^i D_i\chi + ig[\phi, \chi]), \tag{66}$$

$$\delta F_{ij} = -i\xi^C(\gamma_i D_j\chi - \gamma_j D_i\chi), \tag{67}$$

$$\delta D_i\phi = -\xi^C(iD_i\chi^C + [\phi, \chi^C]\gamma^i). \tag{68}$$

Let us write variations (64) and (66) in the form

$$\delta\eta^\alpha = i\xi_\beta L^{\alpha\beta}, \qquad \delta D = \xi_\alpha^C L^\alpha, \tag{69}$$

with $L_{\alpha\beta}$ and L_α appropriately defined. We have factored out g so that a rescaling of all fields $A_i, \phi, D, \eta, \chi \to gA_i, g\phi, gD, g\eta, g\chi$, renders the new $L_{\alpha\beta}$ and L_α g-independent. Lagrangian (59) can be rewritten in the form

$$\mathcal{L} = \frac{1}{g^2}\mathrm{tr}(-L_\beta^\alpha L_\alpha^\beta + \eta_\alpha^C L^\alpha) - \frac{1}{g^2}\varepsilon^{ijk}\mathrm{tr}D_i\phi F_{jk}. \tag{70}$$

Then, in the $g^2 \to 0$ limit, dynamics are governed by configurations which make the supersymmetry variations associated to $L_{\alpha\beta}$ and L_α vanish. That is, configurations satisfying the following first order equations,

$$2D_i\phi = \varepsilon_{ijk}(F^{jk} - 2i\mathbf{C}^{jk}\chi^C\chi), \tag{71}$$

$$D = 0. \tag{72}$$

Concerning the vanishing for the auxiliary field variation

$$\gamma^i D_i\chi + i[\phi, \chi] = 0, \tag{73}$$

it just coincides with the equation of motion for χ, Eq. (62).

Arranging Lagrangian (59) into perfect squares one can see that whenever first order Eqs. (72) and (73) are satisfied, the action coincides with the topological (magnetic) charge. Indeed, starting from (59) one can rewrite the corresponding action in the form

$$S = -\frac{1}{g^2}\mathrm{tr}\int d^3x\left\{\left[\frac{1}{2}\varepsilon_{ijk}(F^{jk} - 2i\mathbf{C}^{jk}\chi^C\chi) - D_i\phi\right]^2 - D^2 + 2i\eta^C(\gamma^i D_i\chi + i[\phi, \chi])\right\} - \frac{1}{g^2}Q_M, \tag{74}$$

where Q_M is a surface term related to the topological charge

$$Q_M = \mathrm{tr}\int dS_i\varepsilon^{ijk}F_{jk}\phi. \tag{75}$$

Note that although we have managed to arrange the action in the form (74), we cannot ensure that configurations satisfying Eqs. (72) and (73) lead to a bound for the action given by the topological charge. This is because the perfect square in the action is not positive definite since $\mathbf{C}_{ij}$ is in general complex and χ in Euclidean space is a Dirac spinor. One can easily see however that any field configuration satisfying (72) and (73) and $\eta = 0$ verifies the equations of motion (60)–(63). Equation (71) can then be seen as the deformed extension of the anti-self-dual BPS equation for the Yang-Mills-Higgs system, the analogous to the deformed Bogomol'nyi Eqs. (42) and (43) for the Abelian-Higgs model.

Let us study solutions to Eqs. (72) and (73). Evidently, the configuration

$$\begin{aligned}
&\phi^a(x) = \phi^{PSa}(x) = \frac{x^a}{r^2}[\mu r\coth(\mu r) - 1] = \frac{x^a}{r}f(r),\\
&\qquad a = 1, 2, 3,\\
&A_i^a(x) = A_i^{PSa}(x) = \varepsilon^{aij}\frac{x_j}{r^2}\left[1 - \frac{\mu r}{\sinh(\mu r)}\right]\\
&\qquad = \varepsilon^{aij}\frac{x_j}{r}(1 - K),\\
&a = 1, 2, 3, \qquad \phi^4(x) = 0, \qquad A_i^4(x) = 0, \qquad \chi = 0,
\end{aligned} \tag{76}$$

where ϕ^{PS} and A_i^{PS} are the well-honored Prasad-Sommerfield [24] monopole SU(2) solutions with μ a constant with mass dimensions solves the first order system. The effects of deformation should arise only if the fermion field $\chi \neq 0$. As done in [11–13] for the instanton case, we shall look for such solutions recursively, starting from (76) and writing

$$\begin{aligned}
&A_i^a(x) = A_i^{PSa}(x) + A_i^{(1)a}(x) + \ldots,\\
&A_i^4(x) = A_i^{(0)4}(x) + \ldots,\\
&\phi^a(x) = \phi^{PSa}(x) + \phi^{(1)a}(x)\ldots,\\
&\phi^4(x) = \phi^{(0)4}(x) + \ldots,\\
&\chi = \chi^{(0)} + \ldots
\end{aligned} \tag{77}$$

Function $\chi^{(0)}$ can be obtained by solving Eq. (73) in the background of a Prasad-Sommerfield monopole. The answer is

$$\chi^{(0)} = D_i\phi^{PS}\gamma^i\zeta, \tag{78}$$

with ζ a constant spinor. One has now to insert this solution in Eq. (71) in order to compute the first order corrections to the gauge and scalar fields. As in the instanton case the bilinear $\chi\chi$ is antisymmetric in the U(2) indices and then the $\mathbf{C}_{\mu\nu}$ perturbation in (30) only affects the U(1) subgroup. Then, SU(2) components of the gauge and scalar fields corrections vanish, $A_i^{(1)a}(x) = 0$, $\phi^{(1)a}(x) = 0$, $a = 1, 2, 3$. Concerning the U(1) sector, one has to solve, for the first order correction, the equation

$$\varepsilon_{ijk}\partial_i\phi^{(0)4} + F_{jk}^{(0)4} = -i\mathbf{C}_{jk}(\chi^{(0)C}\chi^{(0)})^{\{ab\}}\varepsilon_{ab} \equiv \mathbf{C}_{jk}J(x), \tag{79}$$

with $\{ab\}$ indicating antisymmetrization in SU(2) indices. Taking the derivative in both sides one gets for the gauge field (taken in the Lorentz gauge)

$$\nabla^2 A_k^{(0)4} = \mathbf{C}_{jk}\partial_j J. \tag{80}$$

Writing the gauge field in terms of a potential Φ

$$A_k^{(0)4}(x) = \mathbf{C}_{jk}\partial_j\Phi(x), \tag{81}$$

the problem reduces to

$$\mathbf{C}_{jk}\partial_j(\nabla^2\Phi - J) = 0, \tag{82}$$

with

$$J = -i(\chi^{(0)C}\chi^{(0)})^{\{ab\}}\varepsilon_{ab} = -iD^i\phi^{aPS}D^i\phi^{aPS}\zeta^C\zeta. \tag{83}$$

After some calculation one finds that the source J takes the form

$$J = -i\left[\left(\frac{df}{dr}\right)^2 + \frac{1}{r^2}f^2K^2\right]\zeta^C\zeta. \tag{84}$$

With f and K as given in (76) one finally has

$$J = -\left\{\frac{1}{(\mu r)^4} + \frac{2}{\sinh^2(\mu r)}\left[1 - 2\frac{\coth(\mu r)}{\mu r} + \frac{3}{2\sinh^2(\mu r)}\right]\right\}i\zeta^C\zeta. \tag{85}$$

A solution of Eq. (82) can be obtained by solving the Poisson equation

$$\nabla^2\Phi = J. \tag{86}$$

Then, inserting the solution for $A_k^{(0)4}$ in Eq. (79) one finds the solution for $\phi^{(0)4}$.

Since the only correction to the Prasad-Sommerfield solution was the new U(1) components $A_i^{(0)4}$ and $\phi^{(0)4}$, the zero mode equation for χ is not modified and hence the next correction $\chi^{(1)} = 0$. Finally all higher order corrections both for bosonic and fermionic fields vanish.

V. SUMMARY AND DISCUSSION

The connection between self-dual or BPS equations and $\mathcal{N} = 1$ and $\mathcal{N} = 2$ supersymmetry is by now well understood. In this context, studying $\mathcal{N} = 1/2$ supersymmetric models allows to gain some control on relevant aspects of a kind of interpolation towards the $\mathcal{N} = 0$ model. An analysis of instantons solutions in $\mathcal{N} = 1/2$ super Yang-Mills theory was started in [3] and advances on this issue were reported in [11–13]. In this paper we have extended the analysis to the case of solitons and instantons in deformed supersymmetric theories with gauge symmetry breaking. As in the pure super Yang-Mills case, the effect of deformation manifests at the level of the gauge-field–Higgs Lagrangians through the occurrence of a finite number of polynomial terms containing fermion bilinears, both for the Abelian and non-Abelian models. This modifies the surviving supersymmetry transformation law for the gaugino and, consequently, the first order "BPS" equations obtained when one imposes such transformations to vanish.

In the undeformed case, the solution to the first order BPS equations correspond to a bound for the action as can be easily seen by writing the (real) action or energy as a sum of perfect squares plus a topological term—the bound. One can still try to write the deformed action in that way but, being the action in general complex, it has no sense to do this looking for a bound. This has been done for $\mathcal{N} = 1/2$ super Yang-Mills where it was confirmed that configurations satisfying the first order equations arising from the vanishing of SUSY transformations reduce the action to a topological charge [3,11–13]. Concerning the deformed $d = 3$ Yang-Mills-Higgs theory, we have shown here that the same can be done. In contrast, this cannot be achieved for the $\mathcal{N} = 1/2$ supersymmetric Maxwell-Higgs action.

Solutions to the first order BPS equations can in principle be associated with self-dual and anti-self-dual configurations which will be in general differently affected by the deformation. It has been shown in the instanton case [11–13] that anti-self-dual configurations are invariant under the whole $\mathcal{N} = 1/2$ surviving symmetry while self-dual configurations are not. We have shown that the same happens in the monopole case.

In summary, we have shown that no deformed vortex solutions can be found from the first order system except those where all fermions are equal to zero, which reduce to the ordinary Nielsen-Olesen vortices. In the $\mathcal{N} = 1/2$ supersymmetric $d = 3$ Yang-Mills-Higgs case, for which the deformed (complex) action can be written as a sum of squares plus a topological charge, solutions to the first order equations arising from the vanishing of the gaugino supersymmetry variation can be found and they correspond to antimonopole configurations deformed by the nonanticommutativity. We have analyzed these solutions using an iterative process with the deformation parameter $C^{\alpha\beta}$ playing the perturbation parameter. Because of the Grassmann nature of the perturbing fermion field, this iterative procedure stops and in this sense an exact deformed monopole solution can be constructed.

Let us end by mentioning that a connection between the kind of deformation we have discussed and the spectral degeneracy of conventional $\mathcal{N} = 1$ SUSY gluodynamics has been recently discussed in [25]. Remarkably, the analysis in this work suggests that $\mathcal{N} = 1/2$ supersymmetry remains valid for coordinate-dependent $C^{\alpha\beta}$. An analysis of such kind of deformations in supersymmetric gauge-field–Higgs models, extending the one presented here, would then be of interest. We hope to report on this issue in future work.

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APPENDIX

1. The chiral representation

In sections II and III we closely follow Wess-Bagger conventions. An important point about $d = 4$ Euclidean space is that no H.c. relation exists between θ^α and $\bar{\theta}^{\dot{\alpha}}$.

$$\Gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \qquad \sigma^\mu = (i\vec{\sigma}, -1),$$
$$\bar{\sigma}^\mu = (i\vec{\sigma}, 1), \qquad \mathrm{tr}\sigma^\mu\bar{\sigma}^\nu = -2\delta^{\mu\nu}, \tag{A1}$$

$$\Gamma_5 = \Gamma^1 \cdots \Gamma^4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad (\Gamma_5)^2 = 1, \tag{A2}$$

$$C = \Gamma^3\Gamma^1 = \begin{pmatrix} -\epsilon^{\alpha\beta} & 0 \\ 0 & -\epsilon_{\dot{\alpha}\dot{\beta}} \end{pmatrix}, \qquad C^2 = -1,$$
$$C^T = -C, \qquad C\Gamma^m = (C\Gamma^m)^T, \tag{A3}$$

where $\epsilon^{\alpha\beta} = \epsilon_{\dot{\alpha}\dot{\beta}} = -\epsilon_{\alpha\beta} = -\epsilon^{\dot{\alpha}\dot{\beta}} = i\sigma^2$, the σ's have indices $\sigma^m_{\alpha\dot{\alpha}}, \bar{\sigma}^{m\dot{\alpha}\alpha}$. The minimal spinor in $d = 4$ Euclidean space is Dirac (four independent complex components) and in the chiral representation is written in terms of two independent Weyl bi-spinors ψ and $\bar{\psi}$ as we also define

$$\Lambda = \begin{pmatrix} \lambda_\alpha \\ \bar{\lambda}^{\dot{\alpha}} \end{pmatrix}, \tag{A4}$$

$$\Lambda^C = \Lambda^T C = (\lambda^\alpha \bar{\lambda}_{\dot{\alpha}}), \tag{A5}$$

$$P_\pm = \frac{1}{2}(1 \pm \Gamma_5). \tag{A6}$$

The conventions for contracting bi-spinors are

$$\psi\phi = \psi^\alpha\phi_\alpha = \epsilon_{\alpha\beta}\psi^\alpha\phi^\beta, \tag{A7}$$

$$\bar{\xi}\,\bar{\eta} = \bar{\xi}_{\dot{\alpha}}\bar{\eta}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\xi}_{\dot{\alpha}}\bar{\eta}_{\dot{\beta}}. \tag{A8}$$

The deformation of superspace can be rewritten as

$$\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta} = \frac{1}{2}(\sigma_{\mu\nu})^{\alpha\beta}\mathbf{C}^{\mu\nu}, \tag{A9}$$

here $\sigma_{\mu\nu} = \frac{1}{4}(\sigma_\mu\bar{\sigma}_\nu - \sigma_\nu\bar{\sigma}_\mu)$ are anti-self-dual as is $\mathbf{C}^{\mu\nu}$. Recalling that θ_α is the chiral component of the four component Dirac spinor Θ we can rewrite (1) as

$$\{P_+\Theta, \Theta^C P_+\} = \frac{1}{4}P_+\Gamma_{\mu\nu}\mathbf{C}^{\mu\nu}, \tag{A10}$$

where $\Gamma_{\mu\nu} = \frac{1}{2}[\Gamma_\mu, \Gamma_\nu]$. The relations (1) can be stated as

$$\{\Theta, \Theta^C\} = \frac{1}{4}P_+\Gamma_{\mu\nu}\mathbf{C}^{\mu\nu}. \tag{A11}$$

2. Representation for the $d = 4 \to d = 3$ dimensional reduction

In section IV, in order to implement the dimensional reduction from $d = 4$ to $d = 3$ space-time dimensions we use a Gamma matrices representation where

$$\Gamma^i = (i\sigma^i \otimes \sigma^3) = \begin{pmatrix} \gamma^i & 0 \\ 0 & -\gamma^i \end{pmatrix}, \quad \text{for } i = 1, 2, 3, \tag{A12}$$

$$\Gamma^4 = (iI \otimes \sigma^1) = \begin{pmatrix} 0 & iI \\ iI & 0 \end{pmatrix},$$
$$\Gamma^5 = (I \otimes -\sigma^2) = \begin{pmatrix} 0 & iI \\ -iI & 0 \end{pmatrix}, \tag{A13}$$

$$C = (i\sigma^2 \otimes \sigma^3) = \begin{pmatrix} C_3 & 0 \\ 0 & -C_3 \end{pmatrix},$$
$$P_\pm = \frac{1}{2}\begin{pmatrix} I & \pm iI \\ \mp iI & I \end{pmatrix}. \tag{A14}$$

Here γ^i can be identified with $d = 3$ gamma matrices which can be chosen as the Pauli matrices, $\gamma^i = i\sigma^i$, thus leading to

$$C_3 = \gamma^2. \tag{A15}$$

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