Jornadas de Fundamentos de Cuántica

Entanglement and area laws
in weakly correlated gaussian states

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in collaboration with

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Summary

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Typically, Gaussian states are defined as the family of quantum states ρ of a system $\mathcal{S}$ of quantum harmonic oscilators with coordinates $\mathbf{R} = \{\mathbf{Q}_1, \ldots \mathbf{Q}_n, \ldots \mathbf{P}_1, \ldots, \mathbf{P}_n\}$ ($[\mathbf{Q}_j, \mathbf{Q}_k] = [\mathbf{P}_j, \mathbf{P}_k] = 0$, $[\mathbf{Q}_j, \mathbf{P}_k] = \mathbf{i}\delta_{jk}$), such that its Wigner function

$$W(q,p) = \frac{1}{\hbar\pi} \int \langle q + q' | \rho | q - q' \rangle \exp(2ipq') d^n q'$$

is a Gaussian function of its arguments.

In an equivalent way, gaussian states can be defined as such states of the form

$$\rho = \frac{\exp(-\beta \mathbf{H})}{\operatorname{tr} \exp(-\beta \mathbf{H})}$$

for $\mathbf{H}$ a quadratic form in $\mathbf{P}_j$, $\mathbf{Q}_k$

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J.M. Matera
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Gaussian states
Weakly correlated pure gaussian states
Area laws
Results
Conclusions
References

Properties

1. Contains as limit cases the full mixed state $\rho \propto \mathbf{1}$ (for $\beta = 0$), coherent states (for $\beta \to \infty$) as well as complete entangled states (for $\beta \to \infty$ and suitable $\mathbf{H}$).
2. Its evolution $\rho(t) = U(t)\rho(0)U^{\dagger}(t)$ with $U(t) = \exp(-i\mathbf{H}'t)$ is closed if $\mathbf{H}'$ is a quadratic form in $\mathbf{Q}_j$, $\mathbf{P}_j$
3. Satisfie the *Wick Theorem* : the state of each subsystem is also gaussian and is completely determined by the mean value of the local operators $\langle \mathbf{R} \rangle$ and their matrix of second momentums $\Sigma_{\alpha,\beta} = \langle \{\mathbf{R}_\alpha, \mathbf{R}_\beta\}_+ \rangle$

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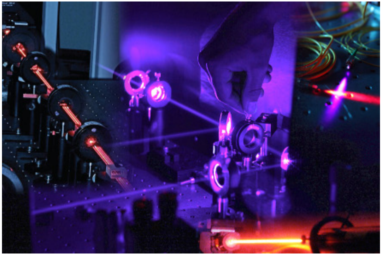
Some applications

Gaussian states had been employed as the zero order modeling states in

1. Quantum Optics
2. Quantum dynamic
3. Quantum Field Theory
4. Quantum Electrodynamics
5. Condensed Matter Systems
6. High Energy Physics



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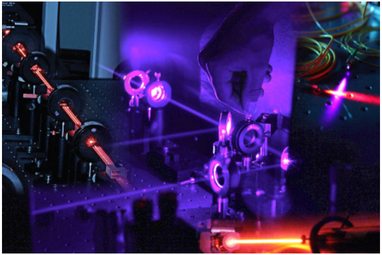

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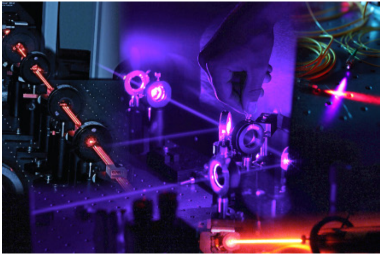

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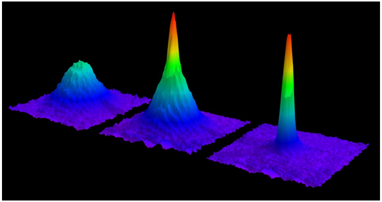

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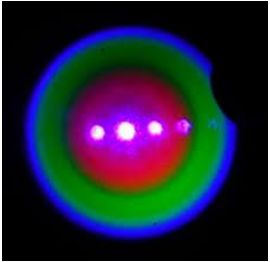

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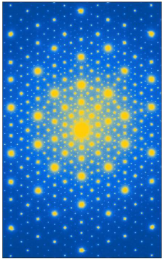

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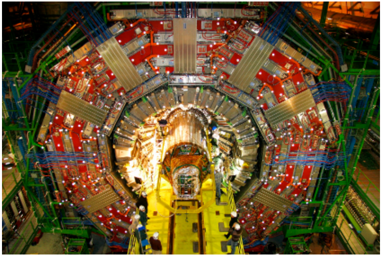

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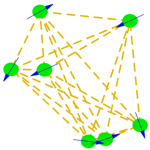

also including systems with finite dimensional Hilbert spaces, like spin s systems through bosonization techniques.

Entanglement in Gaussian states (Fock version)

Gaussian states are defined in terms of the Hilbert space of the observables $\mathbf{Q}_j$ and $\mathbf{P}_j$, which have continuous spectra. However, except for certain limit cases, $\mathbf{H}$ present a discrete spectrum. For this reason, it is convenient to define the operators

$$\mathbf{a}_j = \frac{\mathbf{Q}_j + i\mathbf{P}_j}{\sqrt{2}} \qquad \mathbf{a}_j^\dagger = \frac{\mathbf{Q}_j - i\mathbf{P}_j}{\sqrt{2}} \qquad \mathbf{n}_j = \mathbf{a}_j^\dagger \mathbf{a}_j \quad (1)$$

being $\mathbf{n}_j$ operators with discrete spectrum.

Entanglement in Gaussian states (Fock version)

Because $[\mathbf{n}_i, \mathbf{a}_j] = [\mathbf{a}, \mathbf{n}_i] = \delta_{jk}\mathbf{n}_i$, the action of $\mathbf{a}_j$ and $\mathbf{a}_j^\dagger$ consist into change the quantum number of an eigenstate $|n\rangle$ in one unit:

$$\mathbf{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle \qquad \mathbf{a}|n+1\rangle = \sqrt{n+1}|n+1\rangle$$

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For this reason, these operators are called "ladder operators", which can be understood as operators which create and destroy local "excitations" over an uncorrelated "vacuum" state $|0\rangle = |0\rangle_1 \otimes \ldots \otimes |0\rangle_N$.

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States of the form $\prod_j \frac{(\mathbf{a}_j^\dagger)^{n_j}}{\sqrt{n_j!}}|0\rangle$ which are eigenstates of the local number operators $\mathbf{n}_i$ are known as *Fock states* .

Entanglement in Gaussian states (Fock version)

Correlations in gaussian states are completely determined by its correlation matrix Σ. An equivalent way to encode the same information is in terms of the *Generalized contraction matrix* :

$$\mathcal{D} = \frac{1}{2}\mathcal{U}\Sigma\mathcal{U}^{\dagger} - \mathcal{M} = \langle \mathcal{Z}\mathcal{Z}^{\dagger}\rangle - \mathcal{M} = \begin{pmatrix} F^{+} & F^{-} \\ \bar{F}^{-} & \bar{F}^{+} + \mathbf{1} \end{pmatrix} \quad (2)$$

where $\mathcal{Z} = \left(\mathbf{a}_1, \ldots, \mathbf{a}_n, \mathbf{a}^{\dagger}, \ldots, \mathbf{a}_n^{\dagger},\right)^t = \mathcal{U}\mathbf{R}$

$\mathcal{U} = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathbf{1} & \mathbf{i} \\ \mathbf{1} & -\mathbf{i} \end{pmatrix}$ is a unitary matrix

$\mathcal{M} = \mathcal{Z}\mathcal{Z}^{\dagger} - [(\mathcal{Z}^{\dagger})^{\mathrm{t}}\mathcal{Z}^{\mathrm{t}}]^{\mathrm{t}} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{pmatrix}$ is the symplectic metric and

$F^{+}_{jk} = \langle \mathbf{a}^{\dagger}_k \mathbf{a}_j \rangle_{\rho} \qquad F^{-}_{jk} = \langle \mathbf{a}_k \mathbf{a}_j \rangle_{\rho}$.

For a *pure gaussian state*

$$F^{-}\bar{F}^{-} = F^{+} + (F^{+})^2 \quad (3)$$

Entanglement in Gaussian states (Fock version)

By mean of a canonical linear transformation $\mathcal{Z} = \mathcal{W}\mathcal{Z}'$, (with $\mathcal{W}\mathcal{M}\mathcal{W}^\dagger = \mathbf{1}$), it is possible to bring $\mathcal{D}$ to the diagonal form ($\mathcal{D}'$ with $F^-_{\alpha\alpha'} = 0$ and $F^+_{\alpha\alpha'} = f^\alpha \delta_{\alpha\alpha'}$). The matrix $\mathcal{W}$ can be written in the block form

$$\mathcal{W} = \left(\begin{array}{cc} U & V \\ \bar{V} & \bar{U} \end{array} \right) \quad (4)$$

The first n columns $(u_\alpha, \bar{v}_\alpha)^t$ of $\mathcal{W}$ are the *Symplectic eigenvectors* associated to the *Symplectic eigenvalue* f^α.

Symplectic eigenvectors ψ_α of the matrix $\mathcal{D}$ are the regular eigenvectors of the matrix $\mathcal{D}\mathcal{M}$ with $\psi^\dagger_\alpha \mathcal{M}\psi_\alpha > 0$.

For a *pure gaussian state* $f^\alpha = 0$.

Entanglement in Gaussian states (Fock version)

For a *pure state*, the entanglement between a subsystem $\mathcal{A}$ and its complement $\bar{\mathcal{A}}$ is given by the entropy of any of both subsystems:

$$\mathcal{E}_{\mathcal{A}\bar{\mathcal{A}}} = S_{\mathcal{A}} = S_{\bar{\mathcal{A}}} \tag{5}$$

For *pure gaussian states* this quantity can be expressed in terms of the symplectic eigenvalues of $\mathcal{A}$:

$$S_{\mathcal{A}} = \sum_{\alpha} h(f_{\mathcal{A}}^{\alpha}) \tag{6}$$

where $f_{\mathcal{A}}^{\alpha}$ are the symplectic eigenvalues associated to $\mathcal{D}_{\mathcal{A}}$ (the contraction matrix of the subsystem) and $h(x) = -x\log x + (1+x)\log(x)$ is a convex function.

Logarithmic negativity in Gaussian states (Fock version)

For non pure states or non complementary subsystems $\mathcal{B}$, $\mathcal{C}$, a measure of entanglement is given by the Logarithmic negativity

$$\mathcal{E}^{\mathcal{N}}_{\mathcal{BC}} = \log \|\rho^{\mathrm{t}_{\mathcal{B}}}_{\mathcal{BC}}\|_1 \tag{7}$$

where $\rho^{\mathrm{t}_{\mathcal{B}}}_{\mathcal{BC}}$ is the *partial transposed* density matrix associated to $\rho_{\mathcal{BC}}$ with respect to the subsystem $\mathcal{B}$ and $\|A\|_1 = \mathrm{tr}\sqrt{A^\dagger A}$ is the sum of the absolute values of the eigenvalues of A.

Logarithmic negativity in Gaussian states (Fock version)

For Gaussian states,

$$\mathcal{E}^{\mathcal{N}}_{\mathcal{BC}} = \sum_{\alpha/\tilde{f}^{\alpha}_{\mathcal{BC}}<0} \log(1 + 2\tilde{f}^{\alpha}_{\mathcal{BC}}) \quad (8)$$

where $\tilde{f}^{\alpha}_{\mathcal{BC}}$ are the *negative* symplectic eigenvalues of the contraction matrix $\tilde{\mathcal{D}}_{\mathcal{BC}}$ associated to the density matrix $\rho^{\mathrm{t}_{\mathcal{B}}}_{\mathcal{BC}}$.

As the partial transposition is equivalent in this context to change $\mathbf{a}_k \leftrightarrow \mathbf{a}^{\dagger}_k$ for each k in the subsystem $\mathcal{B}$ and revert its order in each product, $\tilde{\mathcal{D}}_{\mathcal{BC}}$ has blocks $\tilde{F}^{\pm}_{\mathcal{BC}}$ given by

$$\tilde{F}^{\pm}_{\mathcal{BC}} = \begin{pmatrix} \bar{F}^{\pm}_{\mathcal{B}} & \bar{F}^{\mp}_{\mathcal{B},\mathcal{C}} \\ F^{\mp}_{\mathcal{C},\mathcal{B}} & F^{\pm}_{\mathcal{C}} \end{pmatrix} \quad (9)$$

Symplectic eigenvalues in the weakly correlated limit

In the weak correlated limit, relation (3) reduces to

$$F^{+} = F^{-}\bar{F}^{-} + \mathcal{O}^{4}(|F^{-}|_{\infty}) \quad (10)$$

At this order, symplectic eigenvalues coincide with the regular eigenvalues of the matrix

$$F^{+} - F^{-}\bar{F}^{-}u = \lambda u \quad (11)$$

For a pure the state, $F^{+} - F^{-}\bar{F}^{-} = 0$ so, for a given subsystem $\mathcal{A}$,

$$F^{+}_{\mathcal{A}} = F^{-}_{\mathcal{A}}\bar{F}^{-}_{\mathcal{A}} + F_{\mathcal{A},\bar{\mathcal{A}}}\bar{F}^{-}_{\bar{\mathcal{A}},\mathcal{A}} \quad (12)$$

$$\Downarrow$$

at this order, f^{α} are the eigenvalues of the matrix $|F^{-}_{\mathcal{A}\bar{\mathcal{A}}}|^{2} = F^{-}_{\mathcal{A}\bar{\mathcal{A}}}\bar{F}^{-}_{\bar{\mathcal{A}}\mathcal{A}}$, i.e. the square of the *Singular Values* $\sigma^{\alpha}_{\mathcal{A},\bar{\mathcal{A}}}$ of the matrix $F^{-}_{\mathcal{A}\bar{\mathcal{A}}}$.

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$$\mathcal{E}_{\mathcal{A},\bar{\mathcal{A}}} \approx \sum_{\alpha} h\left((\sigma^{\alpha}_{\mathcal{A},\bar{\mathcal{A}}})^{2}\right)$$

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Logarithmic negativity

Negative symplectic eigenvalues $\tilde{f}^{\alpha}$ also are related with the *singular values* of $F^{-}_{\mathcal{A}}$:

$$\tilde{F}^{\pm}_{\mathcal{B},\mathcal{C}} = \begin{pmatrix} \bar{F}^{\pm}_{\mathcal{B}\mathcal{B}} & \bar{F}^{\mp}_{\mathcal{B}\mathcal{C}} \\ F^{\mp}_{\mathcal{C}\mathcal{B}} & F^{\pm}_{\mathcal{C}\mathcal{C}} \end{pmatrix}$$

At leader order, the negative symplectic eigenvalues are the negative eigenvalues of the matrix

$$\tilde{F}^{+}_{\mathcal{B}\mathcal{C}} - \tilde{F}^{-}_{\mathcal{B}\mathcal{C}} \bar{\tilde{F}}^{-}_{\mathcal{B}\mathcal{C}} u = \lambda u$$

Assuming $F^{-}_{\mathcal{B},\mathcal{C}} \gg F^{+}_{\mathcal{B},\mathcal{C}}$ (at least, over certain subspace)

$$\tilde{f}^{\alpha}_{\mathcal{B},\mathcal{C}} \approx -\sigma^{\mathcal{B},\mathcal{C}}_{\alpha} + \frac{(\bar{G}_{\mathcal{B}})_{\alpha\alpha} + (G_{\mathcal{C}})_{\alpha\alpha}}{2}$$

where

$$G_{\mathcal{S}} = \tilde{F}^{+}_{\mathcal{S}} - \tilde{F}^{-}_{\mathcal{S}} \bar{\tilde{F}}^{-}_{\mathcal{S}} \approx \tilde{F}^{-}_{\mathcal{S},\bar{\mathcal{S}}} \bar{\tilde{F}}^{-}_{\bar{\mathcal{S}},\mathcal{S}}$$

i.e. the term $G_{\mathcal{S}}$ takes into account the effect of the environment over the effective entangled modes between $\mathcal{B}$ and $\mathcal{C}$.

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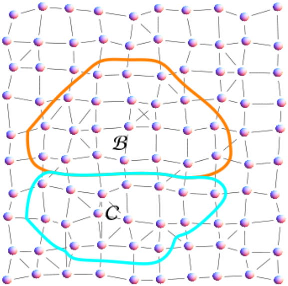
What did we get upto this point?

- A little reduction in the computational requirements.
- A more clear analytical picture of the relationship between entanglement, correlations and influence of the environment.

In the next slides we will see how this picture also give us a geometrical intuition about the entanglement of subsystems: the emergence of *area laws*



What did we get upto this point?

- A little reduction in the computational requirements.
- A more clear analytical picture of the relationship between entanglement, correlations and influence of the environment.

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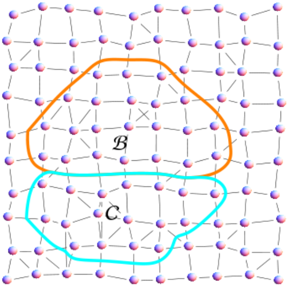

Summary

Why area laws?

In classical thermodynamics, usually we find two classes of quantities:

- Intensive quantities: *independent* of the size of the subsystem considered. (for instance, pressure, densities)
- Extensive quantities: proportional to the *size* of the subsystem considered.

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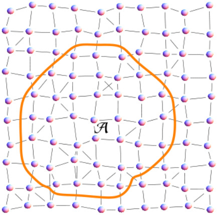
However, in the quantum regime, the *entropy* of certain systems - which in the classical regime is an extensive quantity - acquires a different behaviour, becoming a function of the *area* of the subsystem.

The idea:

- For a pure state, entropy of a subsystem is just entanglement, representing the (non-local) information lost when we can't access to the complementary subsystem.
- For systems with short range interactions (and short range correlations) all the information shared by complementary subsystems belongs to the boundary.
- States which satisfie area laws are easier to be simulated, because entanglement is not too big.
- If a family of states are not able to present area law scaling, quantum correlations can not be well reproduced.



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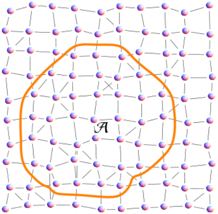

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Gaussian states
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Area laws
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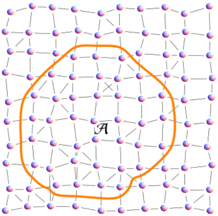

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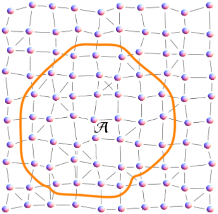

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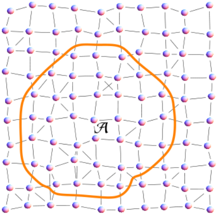

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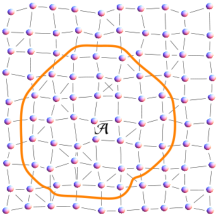

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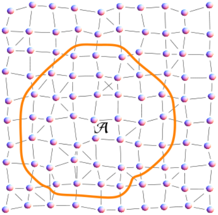

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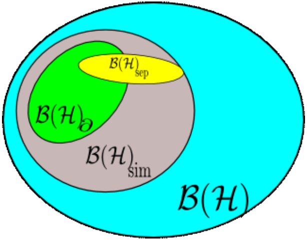
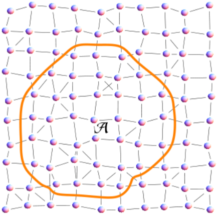


Area law in weakly correlated gaussian states

Now, we will consider the case of a gaussian state where F^- have al its non-null entries of the same order: $F^- \approx f^0 M_{ij}$ where $M_{ij} = 1$ if the site i is correlated with the site j and 0 otherwise.



In this case, singular values of $F^-_{\mathcal{A}\bar{\mathcal{A}}}$ becomes proportional to the singular values of $M_{\mathcal{A}\bar{\mathcal{A}}}$:

$$S_{\mathcal{A}} \approx -(f^0)^2 \log((f^0)^2/e)\mathrm{tr} M_{\mathcal{A}\bar{\mathcal{A}}} M_{\bar{\mathcal{A}}\mathcal{A}} = -f^0 \log f^0/e \sum_k n_k \quad (13)$$

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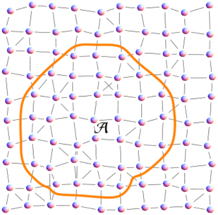

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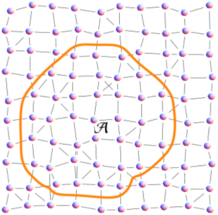

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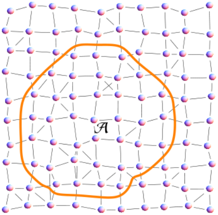

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If each mode in $\bar{\mathcal{A}}$ is correlated with just one mode in $\bar{\mathcal{A}}$

$$\sigma^{\alpha}_{\mathcal{A}\bar{\mathcal{A}}} = f^0 \sqrt{n_k}$$

which leads to the following approximation for the entanglement entropy and negativity

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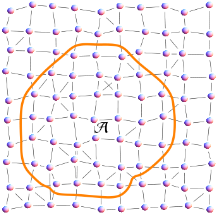

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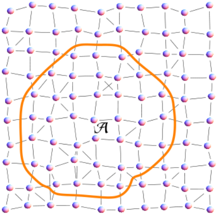

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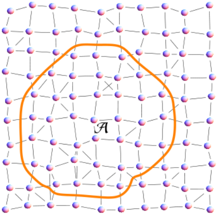

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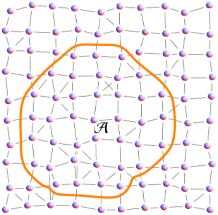
In this sense, $|\partial\mathcal{A}|_1 = \|M\|_1$ and $|\partial\mathcal{A}|_2 = \|M\|_2^2$ define two non equivalent measures of the area of the boundary, which are not necesarily coincident with the euclidean area of any surface bounding the subsystem.



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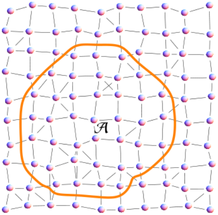

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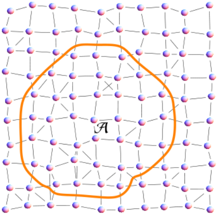
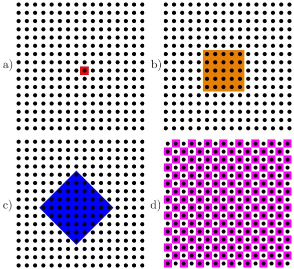


Some examples

This is a comparison for different partitions in the case of a first neighbour correlated square lattice:



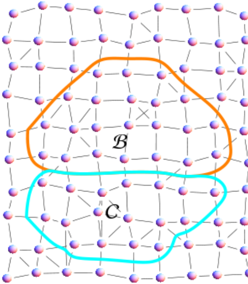
Partition	Euclidean	$\lvert\partial\mathcal{A}\rvert_2$	$\lvert\partial\mathcal{A}\rvert_1$
a)	4	4	2
b)	$4L$	$4(L-2)+8$	$4(L-2)+4\sqrt{2}$
c)	$4\sqrt{2}L$	$8\,(L-2)+12$	$\frac{16}{\pi}L \approx 1.27 \times 4L$
d)	$2n^2$	$2n^2$	$8\,\frac{n^2}{\pi^2} \approx .81n^2$

Area laws in weakly correlated gaussian states

For "bordering" non complementary subsystems $\mathcal{B}$ and $\mathcal{C}$, $\mathcal{E}^{\mathcal{N}}_{\mathcal{BC}}$ can be evaluated as

$$\mathcal{E}^{\mathcal{N}}_{\mathcal{BC}} \approx 2 \log e |\partial\mathcal{B} \cap \partial\mathcal{C}|_1 \qquad (16)$$

which extends the case of complementary subsystems.



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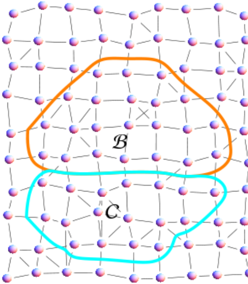

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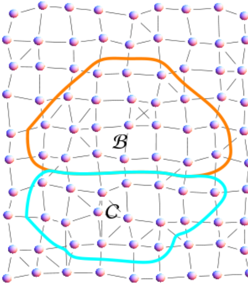

Summary

Entanglement and Log-Negativities in a square lattice

In the next slides, we will consider global states $\rho \propto |0\rangle\langle 0|$ where $|0\rangle$ is the ground state of the Hamiltonian

$$\mathbf{H} = \lambda \sum_{\mathbf{i}} \mathbf{a}_{\mathbf{i}}^{\dagger}\mathbf{a}_{\mathbf{i}} - \sum_{\substack{\mathbf{i},\mathbf{j} \\ |\mathbf{j}-\mathbf{i}|_1=1}} \left[\frac{\Delta^+}{4}\mathbf{a}_{\mathbf{i}}^{\dagger}\mathbf{a}_{\mathbf{j}} + \frac{\Delta^-}{4}(\mathbf{a}_{\mathbf{i}}\mathbf{a}_{\mathbf{j}} + \mathbf{a}_{\mathbf{i}}^{\dagger}\mathbf{a}_{\mathbf{j}}^{\dagger}) \right] \quad (17)$$

for $\mathbf{i}$, $\mathbf{j} \in \mathbb{Z}^2$ the positions of different modes in a square lattice. In particular, we will consider the case of 30×30 lattices with $\Delta^-/\Delta^+ = 2/3$.
These systems are stable for the local energies λ is above $\lambda_c \approx 2(\Delta^+ + \Delta^-)$.

Scaled Entanglement entropy and Log-Negativities for some bipartitions

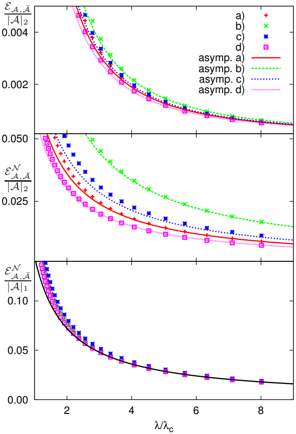

Figure: Scaling of $\mathcal{E}_{\mathcal{A}\bar{\mathcal{A}}}$ and $\mathcal{E}^{\mathcal{N}}_{\mathcal{A}\bar{\mathcal{A}}}$.

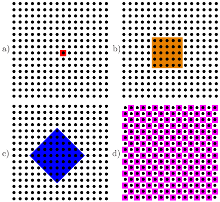

Figure: Different kind of partitions considered.

quant-ph:1211.0581 (2012)

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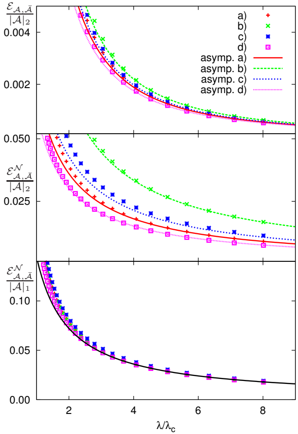

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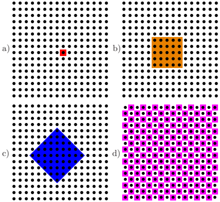

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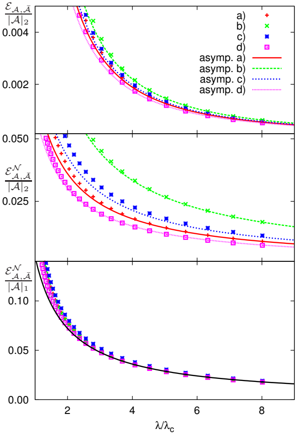

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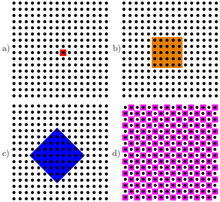

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Some possible non-complementary partitions in a lattice

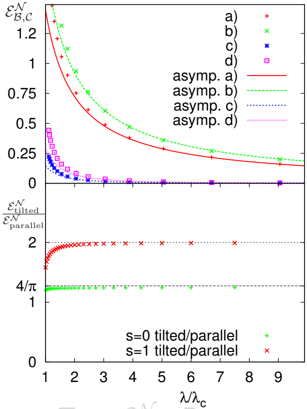
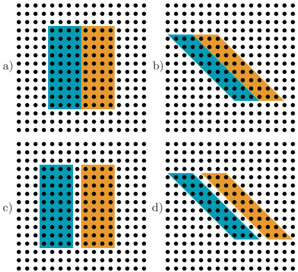


Figure: *Top* : $\mathcal{E}^{\mathcal{N}}_{\mathcal{BC}}$. Bottom: quotient between the correpondent tilted and parallel results for adjacent and 1 mode separated partitions.



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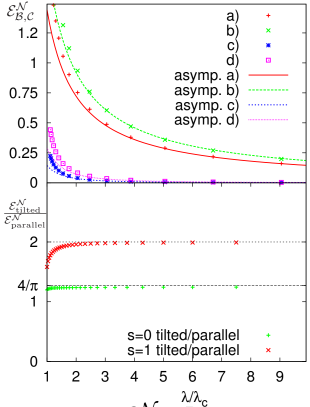

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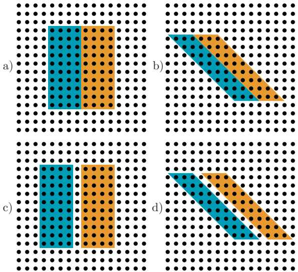

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Some possible non-complementary partitions in a lattice

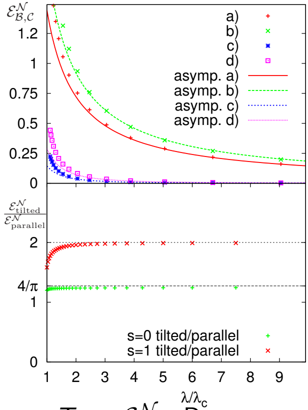

Figure: *Top* : $\mathcal{E}^{\mathcal{N}}_{\mathcal{BC}}$. Bottom: quotient between the correpondent tilted and parallel results for adjacent and 1 mode separated partitions.

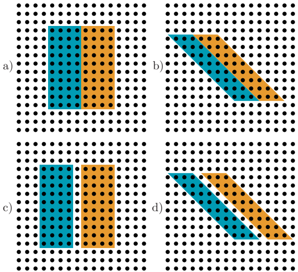

Figure: Different kind of partitions considered.

quant-ph:1211.0581 (2012)

Scale law for the Log-Negativity

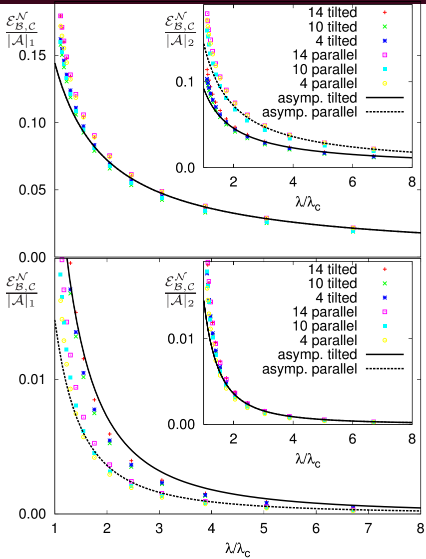

Figure: Scaled Logarithmic Negativity for the previous partitions.

Blocks with different widths and the role of the environment

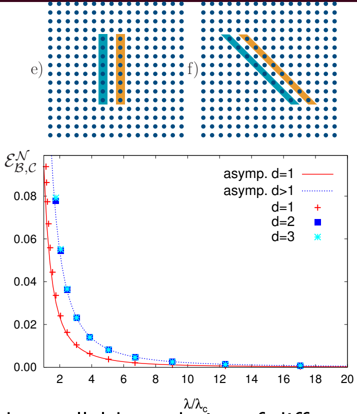

Figure: Blocks with parallel boundaries of different widths and the correspondent log-negativity.

quant-ph:1211.0581 (2012)

Summary

Conclusions

- For weakly correlated gaussian states, the singular values of the contraction matrix $F^{-}_{\mathcal{A}}$ gives an accurate approximation for the exact symplectic eigenvalues of the matrix $\mathcal{D}_{\mathcal{A}}$.
- It allows to evaluate, for pure states, the entanglement between complementary subsystems just in terms of correlations between $\mathcal{A}$ and $\bar{\mathcal{A}}$.
- For the non-pure case, also the symplectic eigenvalues of $\tilde{\mathcal{D}}_{\mathcal{BC}}$ can be evaluated. In this case, the competition between correlations between subsystems and correlations with the environment becomes apparent.
- The formalism also shows the emergence of area laws, giving the right scaling laws for several types of partitions.

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Thanks for your attention.

Summary

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3. **Entrelazamiento cuántico en sistemas de muchos cuerpos**
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4. **Even-odd entanglement in boson and spin systems**
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