Transp Porous Med (2011) 90:779–789
DOI 10.1007/s11242-011-9815-9

A Fractal Model for Predicting Water and Air Permeabilities of Unsaturated Fractured Rocks

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Received: 25 January 2011 / Accepted: 22 July 2011 / Published online: 23 August 2011

Abstract A fractal model to predict water and air permeabilities of unsaturated fractured rocks is presented. The derivation of the model is based on physical and geometric concepts. The pattern of the fracture network is assumed to be fractal and it is described by the Sierpinski carpet. The proposed expressions for the relative water and air permeabilities are closed-form and have five independent parameters: the fractal dimension, the minimum and maximum fracture apertures and the emergence points for water and air flows. The ability of the model to describe experimental data is illustrated by fitting the derived analytical curve to measured data from Grimsel Test Site (Switzerland) and numerical experiments designed by Liu and Bodvarsson (J Hydrol 252:116–125, 2001). In both the cases, the proposed model provides a very good description of water and air permeabilities over several orders of magnitude for the whole range of water saturation.

Keywords Air and water permeabilities · Fractals · Fractured rocks · Two-phase flow

1 Introduction

Fluid flow in hard rocks is often dominated by the highly permeable pathways provided by rock fractures and joints. Immiscible two-phase flow in this type of rocks is of interest to many research problems in groundwater hydrology, like the storage of high-level nuclear wastes in geological formations (Bodvarsson and Tsang 1999). In most practical applications in hydrology and soil sciences, the gaseous phase (air) is assumed to be at a constant pressure (equal to atmospheric) and the system is reduced to the consideration of the water phase only (Bear 1988). This approach is called the Richards approximation, and it has been

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shown a good alternative for describing water flow in the unsaturated zone of the soil, where the influence of the air on the motion of the water is negligible (Celia and Binning 1992). However, the description of gaseous phase flow is of interest in problems like transport of volatile organic compounds or soil remediation by vapor extraction (Sleep and Sykes 1989; Fischer et al. 1998). In these cases, the Richards approximation is no longer valid and a two-phase model must to be used.

Numerical solution of two phase flow equations in unsaturated porous media requires knowledge of constitutive relationships between saturations, permeabilities, and pressures of air and water phases. The experimental determination of permeability relations is tedious and time-consuming and usually the measurements are variable, error-prone, and applicable to only a narrow range of pressures. An alternative to direct measurement is the use of theoretical models which predict water and air permeabilities from the water saturation curve that can be easily measured in laboratory. During the past six decades, several predictive models have appeared in the soil science, engineering and hydrogeology literature. Among the most popular predictive models, we could mention the models proposed by Burdine (1953); Mualem (1976) and Assouline (2001). These models have been developed for describing unsaturated water flow in sedimentary formations (granular porous media). Specific models for fractured hard rocks are virtually nonexistent. Liu and Bodvarsson (2001); Guarracino (2006) and Guarracino and Quintana (2009) have used the Burdine model for predicting water permeability in fractured rocks using different analytical expressions for the water saturation curve. In the Burdine model, pores are described by a group of parallel capillary tubes with different radii, while in Mualem and Assouline models, pore geometries are more complex. In these works, the use of Burdine's model is based on the simplicity of the pore geometry, that allows to describe a single fracture as a linear arrangement of parallel capillary tubes.

In this study, we derive permeability relationships for describing water and air flow in fractured rocks. The fracture pattern is described using the Sierpinski carpet, a classical fractal object that contains a self-similar geometric pattern of pores. Self-similar scaling is a typical property of fractal objects that has been observed in fracture networks by several authors (Turcotte 1986; Obuko and Aki 1987; Barton and Zoback 1992; Berkowitz and Hadad 1997). Sierpinski carpet has been successfully used to describe water content and permeabilities of granular porous media. Based on geometrical properties of the carpet, Tyler and Wheatcraft (1990) obtain water content curves and relate the fractal dimension with the texture of the soil. Yu et al. (2003) derive the permeabilities for a partially saturated porous media consisting of a bundle of tortuous capillary tubes whose radii distribution is described by a Sierpinski carpet. For the particular case of fractured rocks, Guarracino (2006) uses a Sierpinski carpet to derive an analytical closed-form expression for water content. It is important to remark that no field or laboratory data exist to support the use of this type of fractal object to describe fracture patterns. This is the main hypothesis of the model that remains to be experimentally validated. An extensive review on theories, methods, mathematical models and open questions of flow and transport properties in fractal porous media can be found in Berkowitz (2002), Doughty and Karasaki (2002), and Yu (2008).

The proposed permeability model is completely derived from the properties of the Sierpinski carpet, the single-phase flow equation for an individual fracture and classical capillary relations. Water and air permeabilities have closed form analytical expressions with five independent geometrical and physical parameters: the fractal dimension, the minimum and maximum fracture apertures and the emergence points for water and air flows. The fractal model can represent the water permeability values obtained by Liu and Bodvarsson (2001) using numerical simulation techniques and the experimental air permeability values of a fractured crystalline rock at Grimsel Test Site (Switzerland) (Fischer et al. 1998).

The proposed water and air permeability model and the water content expression obtained by Guarracino (2006) are based on the same conceptual model of fractured porous media. The combination of these expressions provides the only set of constitutive relations for fractured hard rocks entirely derived from the geometric properties of the fracture network and physical concepts.

2 Description of the Fracture Network

To derive the expressions for water and air permeabilities, we consider a cube of size a as a representative elementary volume (REV) of the porous medium. The porous medium is conceptualized as an impervious rock matrix with a set of pores defined by a fracture network. Fractures are assumed to be parallel to the flow direction with a spatial distribution pattern that can be described by a Sierpinski carpet. This model is conceptually similar to a classical capillary tube model where the granular porous media is regarded as being equivalent to a bundle of parallel capillary tubes in the flow direction with variable cross sectional areas (Bear 1988). Figure 1 shows an example of two levels of recursion of the fracture pattern obtained with the algorithm described in (Guarracino 2006).

The number l of squares of size x needed to cover the area occupied by fractures of aperture X greater than or equal to x can be expressed as (Mandelbrot 1983):

$$l = \left(\frac{a}{x}\right)^2 - \left(\frac{a}{x}\right)^D, \tag{1}$$

where D is the fractal dimension of the Sierpinski carpet.

For an arbitrary value of l, the cumulative area of fractures whose apertures (X) are greater than or equal to x is given by

$$A(X \geq x) = x^2 l, \tag{2}$$

where x^2 represents the area of an elemental square. Then from Eqs. 1 and 2, we can obtain the following expression for the cumulative area in terms of the parameters of the Sierpinski carpet:

$$A(X \geq x) = a^2(1 - a^{D-2}x^{2-D}). \tag{3}$$

Fig. 1 Two levels of recursion of a Sierpinski carpet with fractal dimension $D = 1.84$. The *fracture pattern* is perpendicular to the flow direction

The area of the carpet covered by fractures whose apertures are within the infinitesimal range x and $x+dx$ is obtained by differentiating Eq. 3 with respect to x:

$$-dA(x) = (2-D)a^D x^{1-D} dx. \tag{4}$$

The negative sign in Eq. 4 implies that the area covered by fractures decreases with the increase of the aperture.

It is important to remark that the Sierpinski carpet represents a geometric pattern which is self-similar at all scales smaller than the initial size of the carpet. The recursion algorithm, if carried to infinity, yields a fracture pattern which is everywhere covered by fractures. However, the fractal behavior typically holds only in a finite range between upper and lower cutoff scales (Acuna and Yortsos 1995). In order to represent a real fractured medium using the Sierpinski carpet, we consider a lower cut-off value $x_{\min}$ and an upper cut-off value $x_{\max}$ for fracture apertures. The upper and lower cut-offs are defined by the largest and smallest apertures observed in the REV, respectively. The lower cut-off is difficult to determine and is usually decided from practical considerations, like the minimum scale of observable resolution (Acuna and Yortsos 1995; Berkowitz 2002).

In the analysis of the fractal dimension of Sierpinski carpet presented in Guarracino (2006) values of D near 2 correspond to lightly fracured media (low density and small apertures) and values near 1 to highly fractured media (high density and large apertures). However, the fractal dimension is not sufficient to describe uniquely the interstitial geometry of a porous medium (Pendleton et al. 2005). In this study, the geometry of a fractured medium is uniquely determined by the fractal dimension of the Sierpinski carpet, the maximum and minimum apertures, and the density of the fractures at the first level of recursion.

3 Relative Permeabilities of a Fractured Rock

In this section, we derive closed-form analytical expressions for air and water relative permeabilities of a fractured rock. The derivation is based on the following general assumptions: (a) fractures are parallel to the flow direction; (b) the cross section of the fractured rock is described by the Sierpinski carpet; (c) Darcy's law is applicable to immiscible two-phase flow; (d) water flow is not coupled with air flow; and (e) the viscosities of the water and air are independent of each other.

The air and water phases contained in the REV are assumed to be subjected to constant effective pressures p_{a} and p_{w}, respectively. Using the classical capillary theory, it is possible to define an effective capillary pressure p from the following equation (Bear 1988):

$$p = p_{\mathrm{a}} - p_{\mathrm{w}} = \frac{2\sigma \cos(\beta)}{x_{\mathrm{ef}}}, \tag{5}$$

where σ is the surface tension, β the contact angle between liquid and solid phases, and x_{ef} an effective fracture aperture. Note that Eq. 5 is only valid under capillary equilibrium. Then, for a given value of p the effective aperture x_{ef} defines a fracture for which liquid and gas phases are in equilibrium, for any other value of x both phases are flowing.

The two-phase flow problem in the fractured porous media can be separated in two single-phase flow problems. We assume that fractures are exclusively occupied by water or air according to the criteria proposed by Pruess and Tsang (1990). The phase occupancy of a fracture with aperture x is governed by the local capillary pressure $p_{\mathrm{c}} = 2\sigma\cos(\beta)/x$. When both water and air phases have access to the fracture, the phase "allowability" will be as follows. For the effective capillary pressure p, the fracture of aperture x will contain water if $p_c > p$ or air if $p_c < p$. This assumption ignores the film flow that takes place on opposite faces of some partially filled fractures. Film flow can be an important mechanism for fast flow in unsaturated fractured rocks and has been included in constitutive models by Or and Tuller (2003). For cases where film flow is significant, the proposed model may not correctly predict the air and water permeabilities. The model also ignores possible effects from water phase which may be held by small-scale roughness or by adsorptive forces in the walls of the fracture (Pruess and Tsang 1990).

From Eq. 5, it follows that all fractures of aperture $x < x_{ef}$ are fully saturated with water while fractures of aperture $x > x_{ef}$ are only occupied by air. The range of fracture apertures in the REV is $x_{min} \le x \le x_{max}$, then Eq. 5 is valid for the range of effective capillary pressures $p_{min} \le p \le p_{max}$ with $p_{min} = 2\sigma\cos(\beta)/x_{max}$ and $p_{max} = 2\sigma\cos(\beta)/x_{min}$. Note that for $p < p_{min}$, all the fractures are fully saturated with water and air flow is zero (single-phase flow in the REV). Contrarily, all fractures are saturated with air when $p > p_{max}$ and water flow is zero.

At the REV scale p_w and p_a are assumed to be constant, then effective pressure gradients are zero and each phase is only driven by gravity. Under this hypothesis, the flow direction is vertical and the flow rate in a single fracture can be obtained by solving the Navier–Stokes' equation (Bear 1988):

$$q_\alpha(x) = \frac{x^2}{12\mu_\alpha}\frac{\partial}{\partial z}(p_\alpha + \rho_\alpha g z) = \frac{\rho_\alpha g x^2}{12\mu_\alpha}; \quad \alpha = \mathrm{a, w} \tag{6}$$

where $\alpha = \mathrm{a, w}$ denotes air and water phases, respectively, z is the vertical coordinate, g is the gravity, μ_α and ρ_α are the dynamic viscosity and the density of the α-phase, respectively.

The total volumetric water flow Q_w through a horizontal cross section of the REV can be obtained by integrating the individual flow rates q_w over the area of fractures occupied by water ($x_{min} \le x \le x_{ef}$):

$$Q_w(x_{ef}) = \int_{x_{min}}^{x_{ef}} q_w(x)\,dA(x) \tag{7}$$

Substituting Eqs. 4 and 6 into Eq. 7, and integrating yields the expression for the total volumetric water flow:

$$Q_w(x_{ef}) = \frac{\rho_w g a^D}{12\mu_w}\frac{D-2}{D-4}\left(x_{ef}^{4-D} - x_{min}^{4-D}\right) \tag{8}$$

Equation 8 can be expressed in terms of the effective capillary pressure using Eq. 5:

$$Q_w(p) = \frac{\rho_w g a^D}{12\mu_w}\frac{D-2}{D-4}(2\sigma\cos(\beta))^{4-D}\left(p^{D-4} - p_{max}^{D-4}\right). \tag{9}$$

On the basis of Buckingham–Darcy's equation (Buckingham 1907), the total volumetric water flow at the REV scale can be expressed as follows

$$Q_w(p) = \frac{k_w(p)a^2}{\mu_w}\frac{\partial}{\partial z}(p_w + \rho_w g z) = \frac{k_w(p)\rho_w g a^2}{\mu_w}, \tag{10}$$

where k_w is the water permeability, and a^2 represents the area of the horizontal cross section of the REV. It is worth to mention that Eq. 10 is valid if the REV contains a dense network of highly interconnected fractures. If the REV can only be defined at a scale similar to the problem of interest, as is the case of poorly connected networks then Eq. 10 is inappropriate (Berkowitz 2002).

Combining Eqs. 9 and 10, we obtain the following expression for the water permeability:

$$k_{\mathrm{w}}(p) = \frac{a^{D-2}}{12}\frac{D-2}{D-4}(2\sigma\cos(\beta))^{4-D}\left(p^{D-4} - p_{\max}^{D-4}\right). \tag{11}$$

The expression for the air permeability as a function of the effective capillary pressure can be derived following the same reasoning over the range of fracture apertures occupied by air ($x_{\mathrm{ef}} \leq x \leq x_{\max}$):

$$k_{\mathrm{a}}(p) = \frac{a^{D-2}}{12}\frac{D-2}{D-4}(2\sigma\cos(\beta))^{4-D}\left(p_{\min}^{D-4} - p^{D-4}\right). \tag{12}$$

The fully saturated permeability or absolute permeability k can be obtained both from Eq. 11 when $p = p_{\min}$ (fully water saturated REV) or from Eq. 12 when $p = p_{\max}$ (fully air saturated REV):

$$k = \frac{a^{D-2}}{12}\frac{D-2}{D-4}(2\sigma\cos(\beta))^{4-D}\left(p_{\min}^{D-4} - p_{\max}^{D-4}\right). \tag{13}$$

The relative permeabilities of water and air, $k_{\mathrm{r,w}}(p)$ and $k_{\mathrm{r,a}}(p)$, are defined as the quotient between Eqs. 11 and 13 and Eqs. 12 and 13, respectively:

$$k_{\mathrm{r,w}}(p) = \frac{k_{\mathrm{w}}(p)}{k} = \frac{p^{D-4} - p_{\max}^{D-4}}{p_{\min}^{D-4} - p_{\max}^{D-4}}, \tag{14}$$

$$k_{\mathrm{r,a}}(p) = \frac{k_{\mathrm{a}}(p)}{k} = \frac{p_{\min}^{D-4} - p^{D-4}}{p_{\min}^{D-4} - p_{\max}^{D-4}}. \tag{15}$$

The above equations are valid for $p_{\min} \leq p \leq p_{\max}$. For values of $p \leq p_{\min}$ all the fractures are fully saturated with water and $k_{\mathrm{r,w}}(p) = 1$, $k_{\mathrm{r,a}}(p) = 0$. On the other hand, all fractures are fully saturated with air for $p \geq p_{\max}$ and $k_{\mathrm{r,w}}(p) = 0$, $k_{\mathrm{r,a}}(p) = 1$.

The relative permeabilities are usually expressed in terms of water saturation S_{w}. The saturation curve for the proposed model of fractured rock was derived in (Guarracino 2006) and is given by:

$$S_{\mathrm{w}}(p) = \frac{p^{D-2} - p_{\max}^{D-2}}{p_{\min}^{D-2} - p_{\max}^{D-2}} = \frac{(2\sigma\cos(\beta)/p)^{2-D} - x_{\min}^{2-D}}{x_{\max}^{2-D} - x_{\min}^{2-D}}. \tag{16}$$

Then by substituting Eq. 16 into Eqs. 14 and 15, we can obtain expressions of $k_{\mathrm{r,w}}(S_{\mathrm{w}})$ and $k_{\mathrm{r,a}}(S_{\mathrm{w}})$:

$$k_{\mathrm{r,w}}(S_{\mathrm{w}}) = \frac{\left[\left(x_{\max}^{2-D} - x_{\min}^{2-D}\right)S_{\mathrm{w}} + x_{\min}^{2-D}\right]^{\frac{4-D}{2-D}} - x_{\min}^{4-D}}{x_{\max}^{4-D} - x_{\min}^{4-D}}, \tag{17}$$

$$k_{\mathrm{r,a}}(S_{\mathrm{w}}) = \frac{x_{\max}^{4-D} - \left[\left(x_{\max}^{2-D} - x_{\min}^{2-D}\right)S_{\mathrm{w}} + x_{\min}^{2-D}\right]^{\frac{4-D}{2-D}}}{x_{\max}^{4-D} - x_{\min}^{4-D}}. \tag{18}$$

Equations 17 and 18 are closed form analytical functions of water saturation S_{w} that depend on the fractal dimension and the minimum and maximum fracture apertures. Both $k_{\mathrm{r,w}}$ and $k_{\mathrm{r,a}}$ are defined over the full range of water saturation $0 \leq S_{\mathrm{w}} \leq 1$. However in real porous media, the water phase becomes disconnected at very low values of S_{w}, and no water flow can be observed through the REV. In contrast, when S_{w} approximates to 1, the air phase becomes disconnected and no air flow can be detected (Kaviany 1995). These facts limit the "effective" water saturation to a narrower range, which can be defined as $0 < S_{w,e} \le S_w < 1$ for water flow and $0 < S_w \le S_{a,e} < 1$ for air flow, where $S_{w,e}$ and $S_{a,e}$ are the emergence points for water and air flows, respectively (Stonestrom and Rubin 1989; Dury et al. 1999). The emergence point of water flow $S_{w,e}$ must be addressed as the value of water saturation above which water phase becomes connected and water flow takes place. Analogously, the emergence point of air flow $S_{a,e}$ must be addressed as the value of water saturation below which air phase becomes connected and air flow is observed.

In order to obtain a more realistic model, we introduce the emergence points and rescale the water saturation S_w of Eqs. 17 and 18 in the following fashion (Fischer et al. 1998; Dury et al. 1999):

$$k_{r,w}(S_w) = \begin{cases} 0 & 0 \le S_w \le S_{w,e} \\ \dfrac{\left[\left(x_{max}^{2-D} - x_{min}^{2-D}\right)\frac{S_w - S_{w,e}}{1-S_{w,e}} + x_{min}^{2-D}\right]^{\frac{4-D}{2-D}} - x_{min}^{4-D}}{x_{max}^{4-D} - x_{min}^{4-D}} & S_{w,e} \le S_w \le 1 \end{cases} \quad (19)$$

$$k_{r,a}(S_w) = \begin{cases} \dfrac{x_{max}^{4-D} - \left[\left(x_{max}^{2-D} - x_{min}^{2-D}\right)\frac{S_w}{S_{a,e}} + x_{min}^{2-D}\right]^{\frac{4-D}{2-D}}}{x_{max}^{4-D} - x_{min}^{4-D}} & 0 \le S_w \le S_{a,e} \\ 0 & S_{a,e} \le S_w \le 1 \end{cases} \quad (20)$$

Note that now $k_{r,w}$ varies from 0 to 1 for the range of water saturation $S_{w,e} \le S_w \le 1$, while $k_{r,a}$ varies from 1 to 0 for $0 \le S_w \le S_{a,e}$.

Figures 2 and 3 show the influence of the fractal dimension D and range of fracture apertures (x_{min}/x_{max}) on relative water permeability $k_{r,w}$. In both the figures, the emergence point for water flow is assumed to be $S_{w,e} = 0.1$. Figure 2 shows that the smaller the fractal dimension, the greater the values of $k_{r,w}$ for the whole range of water saturations. On the other hand, the values of $k_{r,w}$ increase with the decrease of range of fracture apertures (Fig. 3). The influence of x_{min}/x_{max} is significant for small values of water saturation where $k_{r,w}$ can vary several orders of magnitude. Similar behavior is observed for the dependence of relative air permeability $k_{r,a}$ with D and x_{min}/x_{max}.

The water and air permeabilities given by Eqs. 19 and 20, and the water saturation proposed in (Guarracino 2006) provide a constitutive model specifically designed for describ-

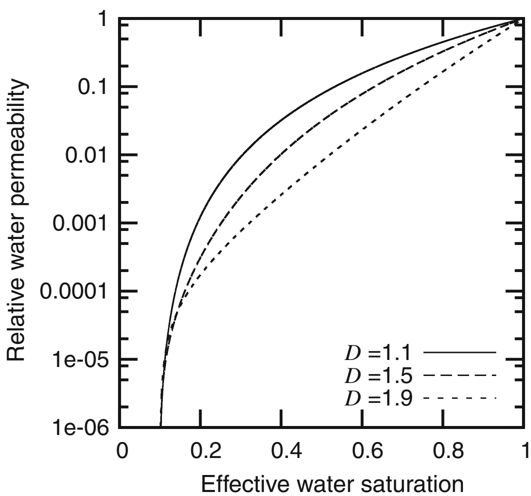
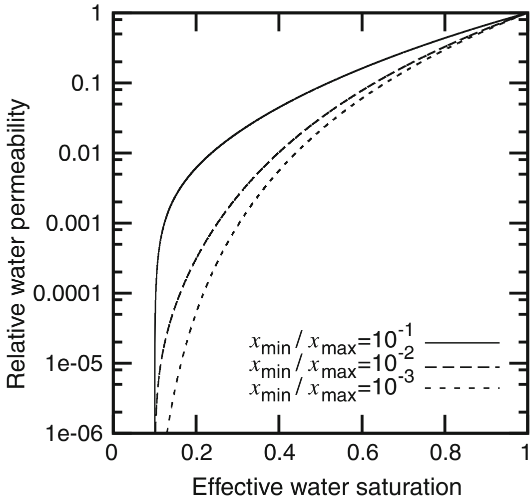


Fig. 2 Relative water permeability curves for three different values of fractal dimension and $x_{min}/x_{max} = 10^{-2}$

Fig. 3 Relative water permeability curves for three different ranges of fracture aperture and $D = 1.5$



ing two-phase flow in fractured rocks. This constitutive model is based on the assumption that the fracture pattern is self-similar and can be described by a Sierpinski carpet. Both $k_{r,w}$ and $k_{r,a}$ have analytical close-forms with five independent parameters with geometrical and physical meaning: D, x_{min}, x_{max}, $S_{w,e}$, and $S_{a,e}$.

4 Comparison with Experimental and Numerical Data

In the present section, we test the ability of the proposed analytical model to reproduce available measured and simulated data from unsaturated fractured rocks. The relative water permeability given by Eq. 19 is tested with numerical experiments designed by Liu and Bodvarsson (2001), while the relative air permeability given by Eq. 20 is tested with experimental data from a fractured crystalline rock at Grimsel Test Site (Gimmi et al. 1997; Fischer et al. 1998).

Direct measurements of water permeability for fractured rocks are particularly difficult to obtain and experimental data for model validation are virtually nonexistent (Liu and Bodvarsson 2001; Tuller and Or 2002). For this reason, the proposed analytical model (Eq. 19) is tested with numerical relative water permeability relations obtained by Liu and Bodvarsson (2001) using a computational procedure which is similar to the laboratory technique of measurement. In their work, the fracture network at REV scale is considered to be a fracture continuum where each fracture is conceptualized as a two-dimensional porous media. For a number of different uniform effective pressures at the REV boundaries, the corresponding values of saturation and relative water permeability are obtained by numerical approximation of Richards' equation. Using this computational procedure Liu and Bodvarsson obtained numerical relations of $k_{r,w}(S_w)$ for two different fractured networks.

The parameters of the proposed model are estimated by fitting the permeability curve (Eq. 19) to the simulated values by Liu and Bodvarsson using an exhaustive search method (Sen and Stoffa 1995). The estimated values of D, x_{min}, x_{max}, $S_{w,e}$ and $S_{a,e}$ for the fracture networks 1 and 2 presented in the paper by Liu and Bodvarsson (2001) are listed in Table 1. Figures 4 and 5 show the comparison, on a logarithmic scale, between the simulated values of

Table 1 Fitted parameters of the proposed model for the tests designed by Liu and Bodvarsson (2001) and for the experimental data at Grimsel Test Site

	Fracture network 1	Fracture network 2	Grimsel Test Site
D	1.468	1.422	1.020
$x_{\min}$ (cm)	0.1836×10^{-4}	0.1249×10^{-4}	0.4701×10^{-3}
$x_{\max}$ (cm)	0.1452	0.1419	0.1556×10^{-2}
$S_{\mathrm{w,e}}$	0.46×10^{-1}	0.82×10^{-1}	–
$S_{\mathrm{a,e}}$	–	–	0.76

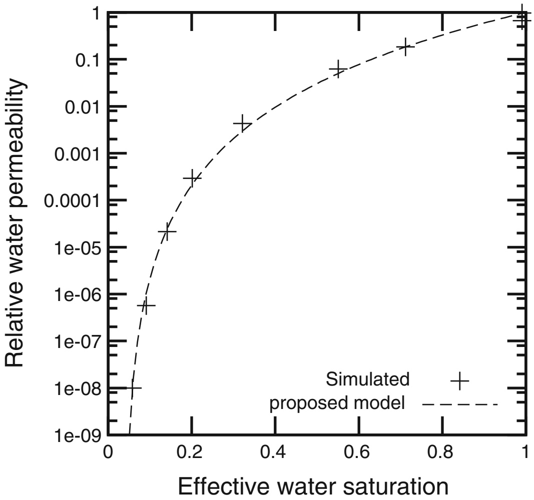


Fig. 4 Comparison between the simulated values of $k_{\mathrm{r,w}}(S_{\mathrm{w}})$ obtained by Liu and Bodvarsson (2001) for fracture network 1 and the predicted values using Eq. 19

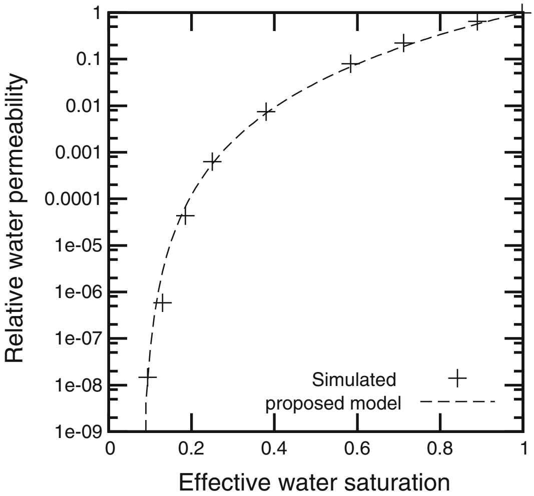


Fig. 5 Comparison between the simulated values of $k_{\mathrm{r,w}}(S_{\mathrm{w}})$ obtained by Liu and Bodvarsson (2001) for fracture network 2 and the predicted values using Eq. 19

$k_{\mathrm{r,w}}(S_{\mathrm{w}})$ and the predicted values using Eq. 19 for fracture networks 1 and 2. In both cases, an excellent agreement between the fitted curve and simulated values is obtained for the whole range of water saturations and over eight orders of magnitude of water permeability.

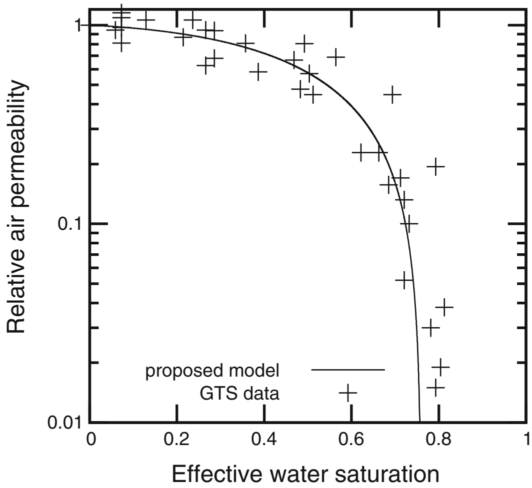


Fig. 6 Comparison between measured data of $k_{r,a}(S_w)$ at Grimsel Test Site (GTS) and the predicted values using Eq. 20

The dataset for a fractured crystalline rock at Grimsel Test Site (Switzerland) reported by researchers of the Swiss Federal Institute of Technology is used to test the proposed model for relative air permeability (Eq. 20). The geological situation of the site is described in detail in Bossart and Mazurek (1991). The rock samples consist of fault gouge with cohesionless material created by brittle deformation (mylonite). For eight rock samples, the relative air permeability as a function of water saturation was measured using a special apparatus designed by Fischer et al. (1998). Gas permeability was determined for dry samples and at various water saturations. Starting with the fully saturated samples, different water saturations were established by evaporation of a certain amount of water.

The values of the different model parameters leading to the best fit of Eq. 20 to the measured data are listed in Table 1. Note that the value of the fractal dimension is very close to 1 indicating the high degree of fracturing of rock samples. Figure 6 shows the comparison between predicted and measured values of $k_{r,a}(S_w)$. The measurements show some scatter, but the proposed model can predict fairly well the air permeability values in the whole range of water saturation.

5 Conclusions

A fractal model for predicting relative air and water permeabilities of fractured rocks has been presented. The geometric pattern of fracture network is described by the Sierpinski carpet. The model has closed-form analytical expressions that are easy to evaluate with five independent parameters: the fractal dimension, the minimum and maximum fracture apertures and the emergence points for water and air flows. The derived relative water permeability curve was tested with the simulated values obtained by Liu and Bodvarsson (2001), while the relative air permeability curve was tested with experimental data from a fractured crystalline rock at Grimsel Test Site. In both the cases, excellent agreements were found for the whole range of water saturation and over several orders of magnitude of air and water permeabilities. Finally, it is important to remark that the combination of the proposed permeability model and the water saturation curve derived by Guarracino (2006) provides a constitutive model specifically designed for air and water flows in unsaturated fractured rocks.

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