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Phenomenological distribution of dark matter halos of galaxy comprising of stellar streams

Mir Hameeda[1,2,3] · Mario C. Rocca[4,5,6]

Abstract Using the potential associated with NFW mass profile (Navarro et al. in Mon. Not. R. Astron. Soc. 402:21, 2010) enclosed at scale radius r_s, we calculate the partition function of the clustering of dark halos comprising of stellar streams, which are the outcome of accretion of globular clusters or dwarf galaxies onto a massive host galaxy. The thermodynamics of the system of halos of stellar streams may provide crucial information on the properties of galaxy clustering. The distribution function is studied through statistical formalism. The study can give us an idea of the extended structure of galaxies and the contribution of halos to the thermodynamics and the mass content of the host galaxy. In an attempt to fit the theoretical orbital velocity to the galaxy rotation curve for the data available for galaxy $M33$, we developed another potential that exactly leads to the best fit orbital velocity of the galaxy. Thus a phenomenological theory for the galaxy with a dark halo is established, with a new density profile.

✉ M.C. Rocca
mariocarlosrocca@gmail.com; rocca@fisica.unlp.edu.ar

M. Hameeda
hme123eda@gmail.com

1   Department of Physics, Government Degree College, Tangmarg, Kashmir, 193402, India

2   School of Physics, Damghan University, P. O. Box 3671641167, Damghan, Iran

3   Inter University Centre for Astronomy and Astrophysics, Pune, India

4   Departamento de Física, Universidad Nacional de La Plata, La Plata, Argentina

5   Departamento de Matemática, Universidad Nacional de La Plata, La Plata, Argentina

6   Consejo Nacional de Investigaciones Científicas y Tecnológicas (IFLP-CCT-CONICET)-C. C. 727, 1900 La Plata, Argentina

1 Introduction

The concept of describing gravity can be well comprehended as an entropic force produced by the changes in the information, when a body moves away from the holographic screen, is proposed in Verlinde (2011). The space-time background is considered to emerge as a mean-field approximation of underlying microscopic degrees of freedom, which was proposed for the first time by Sakharov (1967, 1968, 1991, 2000). The connection between thermodynamics and geometry from relations between heat, entropy, and temperature leads to gravity (Jacobson 1995). The universal nature of gravity is elaborated by the fact that its basic equations have a close resemblance with the laws of hydrodynamics. The laws of gravitation, as derived by Verlinde and the concept of entropic force, opens a new direction to comprehend gravity from the first principles. The Friedmann equations of the dynamical evolution of the FRW universe, from the perspective of entropic force along with the equipartition law of energy and the Unruh temperature, is advocated (Cai et al. 2010; Ling and Wu 2010). Modifications to Newton's law of gravitation, as well as modified Friedmann equations, from the point of view of entropic force, are studied in Sheykhi (2010), Sheykhi and Hendi (2011). Further entropic interpretation of gravity has also been used to discuss the modified Newton's law (Modesto and Randono 1998), the Newtonian gravity as modified in loop quantum gravity (Smolin 2010), the holographic dark energy (Li and Wang 2010; Easson et al. 2011; Danielsson 2010), black hole thermodynamics (Tian and Wu 2010), and the extension to Coulomb force

(Wang 2010). The clustering parameter of galaxies and the effect of extra dimensions on the two-point functions between galaxies in braneworld models is analyzed (Hameeda et al. 2016). The interpretation of the entropic force posses relevance in various contexts (Liu et al. 2010; Kiselev and Timofeev 2010; Konoplya 2010; Banerjee and Majhi 2010; Nicolini 2010; Gao 2010; Myung and Kim 2010; Wei 2010; Easson et al. 2012; Wei et al. 2011).

A well-established fact that there is a deep relation between thermodynamics and gravity is evident from the implication that the Einstein equations can be viewed as a thermodynamic equation of state. Therefore, it is a worthwhile and interesting study to analyze the thermodynamics of the interacting system of dark halos comprising of stellar streams from the entropic force point of view. The regions of galactic halos show behavior similar to a self-interacting classical Boltzmann gas in thermal equilibrium (Davidson et al. 2014). In this study, the gravitational potential of the gas of halos in spherical symmetry is obtained from the circular orbital speed.

In a pioneer work of potential fit to the dark matter halo, two models, triaxial and spherical Navarro Frenk White potential, have been discussed and show best fit to the known dark matter distribution of the simulated halo. Tests have been carried, by selecting stellar streams generated from the cosmological, dark matter N-body simulation Aquarius A (Springel et al. 2008). These tests have been performed through stellar tagging as per the semi-analytic model of star formation (Cooper et al. 2010), and different potential models have been evaluated (Sanderson et al. 2017). The modern cosmological models support the claim that 5/6 of the mass in the Universe is dark matter, which is believed to form the frame on which the galaxies form, evolve, and merge. The power spectrum of matter supports the formation of small objects first and the growth and merging of halos later over time (Wechsler and Tinker 2018). These shreds of evidence suggest that the growth, internal properties, and spatial distribution of galaxies to be closely connected to their counter properties of dark matter halos.

Thus the measurement of galaxies, their clusters, and the distributions capacitates us to map out the corresponding properties of dark matter. For instance, the different clustering properties studied by Peebles (2020), Davis and Peebles (1983), Bahcall and Soneira (1983), Klypin and Kopylov (1983), Kaiser (1984) and the information drawn about the masses of dark matter halos, the masses being the strong factor on which halo clustering depends (Bardeen et al. 1986; Mo and White 1996). N-body simulations shows that dark matter halo growth is function of its mass (Wechlser et al. 2002; Wechsler 2008). Keeping in view the galaxy halo connection, the vice-versa can also be approached. Thus know-

ing the halo properties, one can determine the corresponding properties of galaxies. The paradigm of explaining the clustering of galaxies in space, based on the understanding of the clustering properties of dark matter halos, has shown a considerable emergence over the years. Halo occupation models and abundance matching models have been evolved to know the connection between galaxies and halos at an epoch of time (Conroy and Wechler 2009). The connection between halos implies the average connection between galaxies. The extension of a recent work (Hameeda et al. 2021) is considered, where the observationally supported core halo radii, equivalent to the minimum distance of approach between galaxies interacting gravitationally, is taken as the lower limit for the pertinent configurational integrals. Thus, the configurational integral is evaluated between the two finite limits, and, accordingly, finite results are obtained. Fixing a lower limit by taking it equivalent to halo radii removes the possibility of any divergence arising out of the results (Mir et al. 2021). In the present study, we evaluate the partition function through the statistical formalism (Ahmad et al. 2002, 2006, 2013) and determine the thermodynamic equations of the state of the system of dark halo of the galaxies. We use NFW mass profile of dark halo, with scale radius r_s and the potential defined after avoiding the degeneracies by Kullback–Leibler divergence (KLD) method as discussed (Sanderson et al. 2017).

In this paper, by utilizing the configuration integrals over a spherical volume, we derive the gravitational partition function for dark matter halos, consisting of stellar streams, interacting through modified Newtonian potential, believed to be the best fit potential (Sanderson et al. 2017). The exact equations of state for the cluster of dark matter halo is obtained, by computing Helmholtz free energy, entropy, internal energy, pressure, and chemical potential, which depend on clustering parameter. It is the clustering parameter, which contains complete information about the interacting system of the dark halo of the galaxies.

Further, we emphasize that the general distribution function can give the idea of the distribution of dark halo. We extend the study to the galaxy rotation curve by comparing the theoretical velocities to the data available for the galaxy $M33$. The discrepancies between theoretical results and available data provoke us to explore the best-fit possibilities. While doing so, we developed a phenomenological potential and a new form of density profile for the dark halo of the galaxy. The new results show an excellent fit to the galaxy rotation curve. With the new potential in hand, we could evaluate the thermodynamical equations of state, believing them to be the best fit results.

2 Partition function for the clustering of cosmological dark matter halos

To calculate the partition function, we will follow the procedure given in Ahmad et al. (2002). Thus

$$Z(T, V) = \frac{1}{\Lambda^{3N} N!} \int exp\left(-\left[\sum_{i=1}^{N} \frac{p_i^2}{2m}\right.\right.$$

$$\left.\left. + \Phi(r_1, r_2, r_3, \ldots, r_N)\right]T^{-1}\right)d^{3N}p\, d^{3N}r \quad (2.1)$$

$Z(T, V)$ can be written in the form:

$$Z_N(T, V) = \frac{1}{N!}\left(\frac{2\pi m T}{\Lambda^2}\right)^{3N/2} Q_N(T, V), \quad (2.2)$$

where $Q_N(T, V)$ is the configurational integral and is given as

$$Q_N(T, V) = \int \cdots \int \prod_{1 \le i < j \le N} exp[-\Phi_{ij}(r_1, r_2, \ldots,$$

$$r_N)T^{-1}]d^{3N}r \quad (2.3)$$

The general gravitational potential energy $\Phi(r_1, r_2, \ldots, r_N)$ is considered as a function of the relative position vector $r_{ij} = |r_i - r_j|$ and is expressed as the sum of the potential energies of all pairs. For such gravitational system, the potential energy $\Phi(r_1, r_2, \ldots, r_N)$ is due to all pairs of particles comprising the system. Then,

$$\Phi(r_1, r_2, \ldots, r_N) = \sum_{1 \le i < j \le N} \Phi(r_{ij}) = \sum_{1 \le i < j \le N} \Phi(r) \quad (2.4)$$

Introduce the two particle function

$$f_{ij} = e^{-\Phi_{ij}/T} - 1 \quad (2.5)$$

which vanishes in absence of interactions.

Its form was given in Ahmad et al. (2002). With that form, the configurational integral can be expressed as

$$Q_N(T, V) = \int \cdots \int \left[(1 + f_{12})(1 + f_{13})(1 + f_{23})\right.$$

$$\left. \times (1 + f_{14}) \ldots (1 + f_{N-1,N})\right]$$

$$d^3 r_1 d^3 r_2 \ldots d^3 r_N \quad (2.6)$$

The modeling of gravitational potential has been carried for extended galaxies with halos comprising of stellar streams (Sanderson et al. 2017) and the potential is expressed as

$$\Phi(r) = -G\frac{m_s}{\ln 2 - \frac{1}{2}}\frac{\ln(1 + \frac{r}{r_s})}{r} - \frac{Gm}{(r_{ij}^2 + \epsilon^2)^{1/2}} \quad (2.7)$$

Where ϵ is the softening parameter, which takes care of the extended nature of the galaxy. Assuming $m_s = qm$, where q is a positive number, which gives the idea of amount of mass present in the halo. In present context we will be using $q = 2$. This potential has been considered for enclosed mass at the scale radius r_s of dark matter halo. Scale radius $r_s = \frac{r_{200}}{c} = 15.19$ kpc, with $c = 16.19$ for the halo well fit by an NFW profile (Sanderson et al. 2017). The potential can be used to find the configurational integral and partition function for the cluster of halos of comparable size.

$$f_{ij} = exp\left[\frac{Gm^2}{T(r^2 + \epsilon^2)^{1/2}} + qG\frac{m^2}{\ln 2 - \frac{1}{2}}\frac{\ln(1 + \frac{r}{r_s})}{rT}\right] - 1. \quad (2.8)$$

Assuming moderately dilute systems of halo, the two particle function f_{ij} upon expansion gives

$$f_{ij} = \left[\frac{Gm^2}{T(r^2 + \epsilon^2)^{1/2}} + qG\frac{m^2}{\ln 2 - \frac{1}{2}}\frac{\ln(1 + \frac{r}{r_s})}{rT}\right]. \quad (2.9)$$

Evaluating the configuration integrals over a spherical volume of radius R, gives

$$Q_1(T, V) = V. \quad (2.10)$$

Now, configuration integral $Q_2(T, V)$ can be written as

$$Q_2(T, V) = 4\pi V\left[\int_0^R r^2 dr + \frac{Gm^2}{T}\int_0^R \left(\frac{1}{(r^2 + \epsilon^2)^{1/2}}\right)r^2 dr\right.$$

$$\left. + q\frac{Gm^2}{(\ln 2 - \frac{1}{2})T}\int_0^R \left(\frac{\ln(1 + \frac{r}{r_s})}{r}\right)r^2 dr\right] \quad (2.11)$$

Upon performing integration, this yields

$$Q_2(T, V) = V^2\left(1 + \alpha x\right) \quad (2.12)$$

with $\beta = \frac{3}{2}(Gm^2)^3$ and

$$\alpha = \sqrt{1 + \frac{\epsilon^2}{R^2}} + \frac{\epsilon^2}{R^2}ln\frac{\epsilon/R}{1 + \sqrt{1 + \frac{\epsilon^2}{R^2}}}$$

$$+ q\frac{1}{\ln 2 - \frac{1}{2}}\left(\frac{r_s}{R} - \frac{1}{2} - \left(\frac{r_s^2}{R^2} - 1\right)\ln\left(1 + \frac{R}{r_s}\right)\right) \quad (2.13)$$

Since $R \sim \rho^{-1/3}$, we can write

$$\frac{3}{2}\left(\frac{Gm^2}{RT}\right)^3 = \frac{3}{2}\left(\frac{Gm^2}{T}\right)^3\rho = \beta\rho T^{-3}. \quad (2.14)$$

The expression (2.14) is scale invariant under $\rho \to \lambda^{-3}\rho$, $T \to \lambda^{-1}T$ and $r \to \lambda r$. Thus we can write equation (2.12) as:

$$Q_2(T, V) = V^2(1 + \alpha x), \tag{2.15}$$

where $x = \beta \rho T^{-3}$ and Following similar procedure, we obtain configurational integral for higher orders as:

$$Q_3(T, V) = V^3(1 + \alpha x)^2, \tag{2.16}$$

and

$$Q_4(T, V) = V^4(1 + \alpha x)^3. \tag{2.17}$$

Thus, for most general case, we have

$$Q_N(T, V) = V^N(1 + \alpha x)^{N-1} \tag{2.18}$$

Hence, the gravitational partition function is obtained explicitly by substituting the value of $Q_N(T, V)$ given in (2.18) in (2.2) as,

$$Z_N(T, V) = \frac{1}{N!}\left(\frac{2\pi m T}{\lambda^2}\right)^{3N/2} V^N(1 + \alpha x)^{N-1}. \tag{2.19}$$

3 Thermodynamic implications of dark matter halo

We compute below the various thermodynamic quantities relevant for interacting system of halos interacting through a best fit potential. It is well-known that the thermodynamic quantities can be easily calculated from the gravitational partition function. For example, Helmholtz free energy, defined, generally, by $F = -T \ln Z_N(T, V)$ is calculated as

$$F = -T ln\left(\frac{1}{N!}\left(\frac{2\pi m T}{\lambda^2}\right)^{3N/2} V^N(1 + \alpha x)^{N-1}\right) \tag{3.1}$$

Now, it is easy to compute entropy S for a given Helmholtz free energy with formula, $S = -\left(\frac{\partial F}{\partial T}\right)_{N,V}$. Here entropy reads

$$S = N ln(\rho^{-1} T^{3/2}) + (N - 1) ln(1 + \alpha x) - 3N \frac{\alpha x}{1 + \alpha x}$$
$$+ \frac{5}{2}N + \frac{3}{2}N ln\left(\frac{2\pi m}{\lambda^2}\right). \tag{3.2}$$

For large N such that $N - 1 \approx N$, this can further be simplified as

$$\frac{S}{N} = ln(\rho^{-1} T^{3/2}) + ln(1 + \alpha x) - 3b + \frac{S_0}{N}, \tag{3.3}$$

where the expression for $b = \frac{\alpha x}{1 + \alpha x}$, is the clustering parameter that contains the complete information of the clustering (Saslaw and Hamilton 1984). Clustering parameter b depends on α, which actually comes from the modifications of

interacting potential such as ϵ, the softening parameter taking care of the extended nature of the galaxy, and r_s, the halo radius, and definition $S_0 = \frac{5}{2}N + \frac{3}{2}N ln\left(\frac{2\pi m}{\lambda^2}\right)$ are utilized.

Employing expression (3.1) and (3.2), the internal energy, defined as $U = F + TS$, for a system of halos is calculated by

$$U = \frac{3}{2}NT(1 - 2b). \tag{3.4}$$

It is evident from the above expression that the internal energy depends on the clustering parameter explicitly.

The pressure and chemical potential, utilizing the standard notations and definitions $P = -\left(\frac{\partial F}{\partial V}\right)_{N,T}$ and $\mu = \left(\frac{\partial F}{\partial N}\right)_{V,T}$ respectively, are calculated by

$$P = \frac{NT}{V}(1 - b) \tag{3.5}$$

$$e^{\frac{N\mu}{T}} = \left(\frac{\bar{N}}{V}T^{-3/2}\right)^N (1 + \alpha x)^{-N} e^{-Nb}\left(\frac{2\pi m}{\Lambda^2}\right)^{-3N/2} \tag{3.6}$$

Here we observe that the pressure and chemical potential depends explicitly on the parameter α explicitly.

4 General distribution function for dark halo galaxies

The definition of grand partition function is,

$$Z_G(T, V, z) = \sum_{N=0}^{\infty} z^N Z_N(V, T) \tag{4.1}$$

where z is the activity. The grand partition function for our gravitationally interacting system of halos is calculated by

$$ln Z_G = \frac{PV}{T} = \bar{N}(1 - b) \tag{4.2}$$

where the expression (3.5) is utilized.

Now, the probability of finding N particles in volume V can be estimated by relation

$$F(N) = \frac{\sum_i e^{\frac{N\mu}{T}} e^{\frac{-U}{T}}}{Z_G(T, V, z)} = \frac{e^{\frac{N\mu}{T}} Z_N(V, T)}{Z_G(T, V, z)}, \tag{4.3}$$

with the help of (4.2), the distribution function is computed, precisely Thus, the distribution function becomes

$$F(N) = \frac{\bar{N}^N}{N!} e^{[-Nb - \bar{N}(1-b)]}. \tag{4.4}$$

$$b = \frac{\alpha x}{1 + \alpha x} \tag{4.5}$$

5 Rotation curve

The dynamics of galaxies, especially their rotation curves, provide the main observational clue about the existence of dark matter (de Almeida et al. 2018). The dark matter problem, though, has been addressed by alternative theories of gravity other than Newtonian at galactic scales (Finzi 1963; Milgrom 1983; de Blok and McGaugh 1998; Sanders and McGaugh 2002; Milgrom 2006; Moffat 2006; Rodrigues et al. 2010; Brownstein and Moffat 2006; Capozziello et al. 2007; Frigerio Martins and Salucci 2007; Banados 2008; Mannheim and O'Brien 2012). In one of the pioneering works, dark matter is included by assuming that the fifth force couples to dark matter and baryons. The work includes baryonic gas, disk, and bulge components, along with an NFW halo of dark matter (de Almeida et al. 2018). In our present attempt, we model the gravitational potential for extended galaxies with halos comprising of stellar streams (Sanderson et al. 2017) and use the potential to obtain the orbital speed using the relation

$$\frac{d\phi}{dr} = \frac{v^2}{r} \tag{5.1}$$

Where v is the orbital speed comprising of two contributions v_g, the orbital speed of galaxy, and v_d, the corresponding orbital speed of dark halo. Thus, we can write

$$v^2 = v_g^2 + v_d^2 \tag{5.2}$$

$$v^2 = \frac{Gmr}{(\epsilon^2 + r^2)^{\frac{3}{2}}} - qG\frac{m}{\ln 2 - \frac{1}{2}}\frac{r - (r_s + r)\ln(1 + \frac{r}{r_s})}{r(r_s + r)} \tag{5.3}$$

$$v^2 = \frac{Gm}{r}\left[\left(1 + \frac{\epsilon^2}{r^2}\right)^{-3/2} + q(\ln 2 - 1/2)^{-1}\right.$$
$$\left. \times \left[\left(\ln(1 + \frac{r}{r_s})\right) - \left(1 + \frac{r_s}{r}\right)^{-1}\right]\right] \tag{5.4}$$

The speed obtained when compared to the rotation curve data of galaxy $M33$ shows a considerable deviation, thus prompts one to think beyond the given potential. The potential we used in the present case contains a contribution from both the extended structure of the galaxy and the dark matter content of the halos, but the discrepancy is still there. Thus, we extend the analysis in the next section to treat the potential in a phenomenological manner with expectations of good results.

6 New potential

On analyzing, we find that the velocity obtained in equation (5.4) is not appropriate to construct the velocity curve for
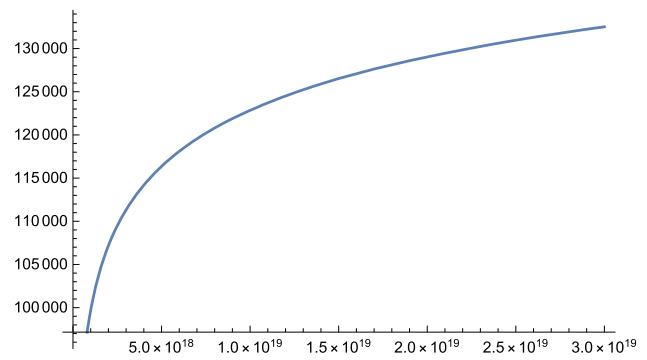

Fig. 1 Curve of velocities. On the x axis the magnitude is given in meters (m) and on the y axis the velocity is given in m/s

the galaxy M33. However, it turns out to be a good guide for our purpose. In a sort of hit and trial effort, we found that by multiplying the mentioned equation by a mere factor of $(20h)^2 mr$, we get the appropriate speed, given by

$$v^2 = \frac{[20(h/[h])]^2 Gm^2}{[r][m]}\left[1 + q(\ln 2 - 1/2)^{-1}\right.$$
$$\left. \times \left[\left(\ln(1 + \frac{r}{r_s})\right) - \left(1 + \frac{r_s}{r}\right)^{-1}\right]\right] \tag{6.1}$$

Where $q = m_s/m$, m_s being the mass of dark halo. The speed thus obtained lead to the corresponding gravitational potential of the form (here, we have taken point size approximation i.e. $\epsilon = 0$)

$$\phi(r) = \frac{[20(h/[h])]^2 Gm^2}{[r][m]}\left[\ln r - q(\ln 2 - 1/2)^{-1}\right.$$
$$\left. \times \left(\ln(r + r_s) + Li_2(-\frac{r}{r_s})\right)\right] \tag{6.2}$$

Where Li_2 is dilogarithm distribution.

On plotting the results in Figure 1, we find that the velocity curve obtained is in complete agreement with the experimental curve of the galaxy M33, which indeed is a surprising result. This indicates that the combo of dark matter halo and the modified gravity is a potential candidate to treat the astrophysical discrepancies like that of the galaxy rotation curve. The study will be given deep attention in our future endeavors. It should be taken into account that the experimental error in the speed measurement is of the order of 20%. Notice that the data in our curve matches, taking into account the error in them, with the data given in the tables of the references Mundase (2019) and Kam et al. (2017).

Figure 2 shows the velocity curve corresponding to formula (5.4). There are no coincidence with experimental results. Figure 3 shows the potential given in Eq. (6.2). Figure 4 shows density corresponding to Eq. (7.2).
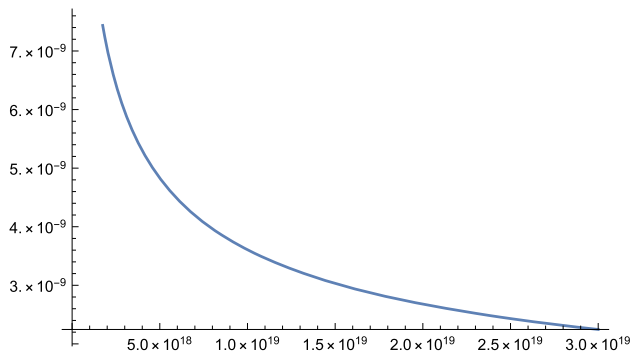
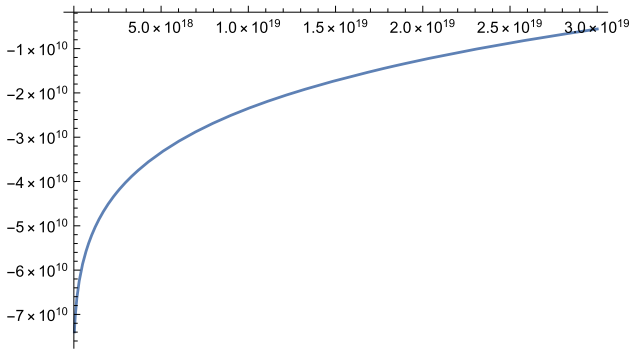**Fig. 2** Curve of velocities corresponding to Eq. (5.4)

Fig. 3 Potential corresponding to (6.2). Note that the potential is negative and tends to zero with increasing distance, as it should be
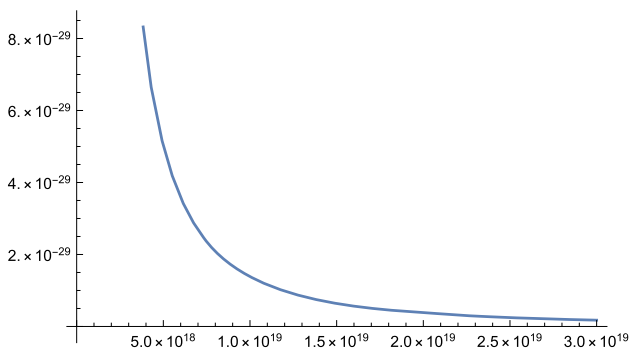

Fig. 4 Density corresponding to Eq. (7.2). Notice that the density decreases with distance, as it should be

7 New density profile

With the expectation that the new potential obtained is the near right form of potential for the large-scale structure of the universe, we believe that the density profile of the galaxies should be different than the NFW-profile.

Thus, using the Poisson equation, we find that the phenomenological development leads to the new density profile for the galaxy with a dark halo.

From (6.1)

$$\triangle\phi(r) = 4\pi G\rho(r) \tag{7.1}$$

We then obtain:

$$\rho(r) = \frac{[20(h/[h])]^2 m^2}{4\pi r^2 [r][m]} \left\{ 1 + q(\ln 2 - 1/2)^{-1} \left[ln\left(1 + \frac{r}{r_s}\right) \right. \right.$$
$$\left. \left. - \frac{r}{r + r_s} + \frac{r^2}{(r + r_s)^2} \right] \right\} \tag{7.2}$$

Further insight is needed in this study, which obviously we reserve for our upcoming works. We may extend the approach to other modified gravities in presence of the dark halos.

8 New partition function

In a similar way, as followed in the earlier section, we now use the new potential to study the large-scale structure formation using statistical mechanics and develop the thermodynamic equations of state. The important thing about the study is that the new potential we obtained fits the experimental data of the galaxy rotation curve, is thus expected to provide better results in the form of the partition function, distribution function, and the thermodynamic properties of the system.

Thus the new partition function corresponding to the new potential, considered as best fit for the galaxy $M33$ is:

$$Q_2(T, V) = V^2(1 + \alpha_h y), \tag{8.1}$$

where:

$$\alpha_h = q(\ln 2 - 1/2)^{-1}$$
$$\times \left[\frac{4}{3}\left(1 + \frac{r_s^3}{R^3}\right)\ln(R + r_s) + Li_2\left(-\frac{R}{r_s}\right) - \right.$$
$$\left. \left(\frac{1}{3} + \frac{5r_s^3}{3R^3}\right)\ln r_s + \frac{2r_s}{3R} - \frac{4r_s^2}{3R^2} - \frac{1}{3} \right] - ln R + \frac{1}{3} \tag{8.2}$$

and

$$y = \frac{[20(h/[h])]^2 Gm^2}{[r][m]T} \tag{8.3}$$

Following similar procedure, we obtain configurational integral for higher orders as:

$$Q_3(T, V) = V^3(1 + \alpha_h y)^2, \tag{8.4}$$

and
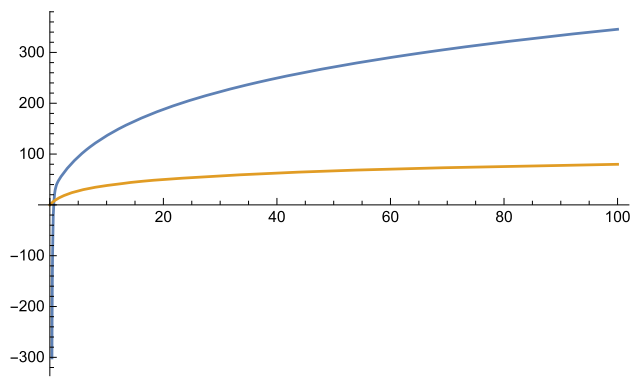
$$Q_4(T, V) = V^4(1 + \alpha_h y)^3. \tag{8.5}$$

Fig. 5 Comparison of α's blue curve corresponds to (8.2) and yellow curve corresponds to (2.13)

Thus, for most general case, we have

$$Q_N(T, V) = V^N (1 + \alpha_h y)^{N-1} \tag{8.6}$$

Hence, the gravitational partition function is obtained explicitly by substituting the value of $Q_N(T, V)$ given in (2.18) in (2.2) as,

$$Z_N(T, V) = \frac{1}{N!} \left(\frac{2\pi m T}{\lambda^2} \right)^{3N/2} V^N (1 + \alpha_h y)^{N-1}. \tag{8.7}$$

The partition function obtained can lead to different thermodynamic properties, which can be treated as the best fit thermodynamic properties of galaxies with dark halos. The thermodynamic equations will have the same form, with only replacement of α by α_h. This will lead to a new clustering parameter, which contains whole information of clustering. Thus we concentrate on the comparative analysis of two parameters. Figure 5 shows the comparison.

9 Discussion

In this paper, we have considered an interacting system of dark halos galaxies comprising of stellar streams in the expanding universe and evaluated the gravitational partition function for dark halos interacting through modified Newtonian potential by utilizing the configuration integrals over a spherical volume. The general method followed is the usual statistical mechanics and thermodynamics. We have computed various thermodynamical quantities, for instance, Helmholtz free energy, entropy, internal energy, pressure, and chemical potential, which depend on clustering parameter explicitly, to study the exact equations of state for dark halos. Further, we have studied the general distribution function for such a system.

We extended the study to the galaxy rotation curve and compared the theoretical velocity to the data available for the galaxy $M33$. The curve showed a discrepancy with the

theoretical results obtained from the potential. In order to explore the possibility of reducing the discrepancy, we, in a hit and trial effort, developed a phenomenological potential and the corresponding density profile which exactly fits the data. The corresponding partition function for the large-scale structure formation is also obtained using the new potential and the new thermodynamic properties obtained depend on the parameter α_h, which is analogous to the earlier parameter α. The α_h actually carries the impact of the new potential and density profile of the dark matter halo of the galaxy and is reflected in all the properties. We have taken a comparative glimpse of the α_h with that of the earlier parameter α. The deviation is clearly visible from Figure 5. The increased level of α_h and the best fit of rotation curve with that of the new potential provides a clue that a combo of dark halo and a specifically modified form of a potential alternative to Newtonian can be the potential candidates to solve some astrophysical and cosmological mysteries of the large scale universe.

In recent work (Mir et al. 2021), we, in an analytical attempt, solved the configurational integral with a lower limit of integral equal to that of the halo radii; the operation removed the singularities and provided divergence-free partition function with finite thermodynamic properties; thus, the presence of halo not only removes mathematical difficulty (Mir et al. 2021) but also gives best fits for galactic rotation curve in our present study. In the present study inclusion of extended structure and halo radii removes the singularity of the integral. The disappearance of the divergences, in theory, is indeed an achievement. The extended structure and the presence of a dark halo made the calculations easier, with no need to regularize the theory, using complicated techniques to remove the infinities. And, additionally, the calculation of thermodynamic variables is simplified considerably and turns out to be all finite and free of singularities due to the presence of terms ϵ and the halo radius, which is important progress in the theory of gravity. The main parameter of the theory is the clustering parameter, which depends on the α_h and the α_h itself contains the information of the interacting potential involved. It is the clustering parameter that enters into all the thermodynamic properties and modifies the equations of state. This actually contains the complete information on the clustering of galaxies and halos. The clustering parameter is also connected to the correlation function, and it is the correlation function that is important for determining the galaxy distribution (Saslaw and Hamilton 1984; Ahmad et al. 2013; Farooq and Hameeda 2010). It is available in the literature about galaxy clustering that the clustering parameter is closely related to the number variance of the distribution function of galaxies, and this is also an important parameter for observational cosmology. A deep and detailed mathematical and theoretical insight is needed to explore the best alternatives of gravity in the presence of the galaxy halos.

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