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RESEARCH ARTICLE - SOLID EARTH SCIENCES

Validation of recent geopotential models in Tierra Del Fuego

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Abstract This work presents a validation study of global geopotential models (GGM) in the region of Fagnano Lake, located in the southern Andes. This is an excellent area for this type of validation because it is surrounded by the Andes Mountains, and there is no terrestrial gravity or GNSS/levelling data. However, there are mean lake level (MLL) observations, and its surface is assumed to be almost equipotential. Furthermore, in this article, we propose improved geoid solutions through the Residual Terrain Modelling (RTM) approach. Using a global geopotential model, the results achieved allow us to conclude that it is possible to use this technique to extend an existing geoid model to those regions that lack any information (neither gravimetric nor GNSS/levelling observations). As GGMs have evolved, our results have improved progressively. While the validation of EGM2008 with MLL data shows a standard deviation of 35 cm, GOCO05C shows a deviation of 13 cm, similar to the results obtained on land.

Keywords Global geopotential models · RTM data · Tierra del Fuego · GOCE · EGM2008

Introduction

With the advent of the GOCE (Gravity field and steady-state Ocean Circulation Explorer) and GRACE (Gravity Recovery and Climate Experiment) dedicated gravity satellite missions, global geopotential models (GGMs) started to be regularly produced. Some are satellite-only models, like GO_CONS_GCF_2_TIM_R5 (Brockmann et al. 2014), GOCO05 s (Mayer-Gürr, T. and the GOCO Team 2015), andEIGEN-6S4v2 (Förste et al. 2016), with a spherical harmonic expansion of the gravity field up to degree 180–200. There are also combined models that use satellite data in combination with terrestrial data. Some notable examples include EGM96 (Lemoine et al. 1998), EGM2008 (Pavlis et al. 2012), the EIGEN family (Förste et al. 2013, 2014, among others), and the most recent GOCO05C (Fecher et al. 2017).

The availability of so many GGMs often presents a challenge to the user regarding which model to use. Therefore, the regional validation of GGMs plays an important role, since their performance is not homogeneous over the entire planet. Erol (2012) indicates that this is a way to provide reliable information on the GGMs' performance and their adjustment to the local gravity field.

The error of the GGMs can be divided into two types: commission and omission errors (Jekely 2009). The former is related to the quality of the spherical harmonic coefficient determination, which depends on the quality of the input data. The latter is caused by the truncation of the spherical harmonics' expansion up to a maximum degree. This truncation is related to data distribution which has a limited spatial resolution (Torge and Müller 2012). The quality and distribution of the data sets used in a GGM solution (especially terrestrial gravity) constrain the accuracy of any gravity field functional computed via a spherical harmonic synthesis.

GGMs are commonly examined according to different methods: through their coefficients and formal error degree variances, comparison against a reference GGM, and by means of external data such as GNSS/levelling (Tsoulis

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and Patlakis 2013; Ustun and Abbak 2010; Vergos et al. 2006).

When the evaluation of GGMs comprises observed data such as GNSS/levelling, it is necessary to take into account that they contain the full gravity spectrum, while GGM geoid heights or height anomalies contain the gravity spectrum up to a certain degree and order.

This paper presents a case study of a small region, located in the southern part of Tierra del Fuego Island, which is notable for the absence of terrestrial data included in any GGM. Since mean lake level (MLL) data exists for the region, it constitutes a valuable test area for the evaluation of GGMs. This information was derived from GPS buoys and simultaneous pressure tide gauge observations (Del Cogliano et al. 2007) on the Fagnano Lake, which are not included as input data in any GGM. Therefore, they can be considered as independent data. In addition, there is no terrestrial gravity data available for the lake surroundings because this region is part of the southern Andes, and it is characterised by the absence of roads and on-land access. Thus, as explained in Gomez et al. (2013) these MLL measurements represent the only geodetic information available to validate GGMs in the southern part of Tierra del Fuego.

The aim of this work is to validate and examine some of the latest combined GGMs to reveal their improvements in the modelling of the gravitational field. The selected models are: EGM2008, EIGEN6C4, GECO, GGM05c, and GOCO05C.

The comparisons will be made against the height anomalies derived from GNSS/levelling points and the previously mentioned MLL data.

Previous validations of the GOCO05C yielded the following RMS between geoid heights derived from the model and GNSS/levelling data for Australia, Germany, Brazil, and the USA: 24, 4, 30, and 58 cm, respectively (Fecher et al. 2017). Similar results have been obtained when evaluating EGM2008 and EIGEN6C4 in the countries mentioned above, except for the USA, which shows an RMS of 24.5 cm (Förste et al. 2014).

In the case of Argentina, the validation of EGM2008 on GNSS/levelling points indicates an RMS of 31 cm, while EIGEN6C4 yields 28 cm, when the 2016 levelling network adjustment is considered (Piñon 2016).

For previous GGMs, including EGM2008, 15–10 cm RMS were obtained when evaluating on GNSS/levelling points located in the Argentine portion of Tierra del Fuego (Tocho 2012; Gomez et al. 2014).

The omission error can be estimated by a high degree GGM like EGM2008, or by the combination of a GGM with Residual Terrain Modelling (RTM) data to take into account the short frequency of the Earth's gravity field spectrum (Hirt et al. 2010). Improvements are not expected in all the tested cases since, according to Ferreira et al. (2013), GGMs do not always show a significant improvement at the shortest wavelengths of the gravity field. They depend not only on the local gravity data considered to develop the GGMs, but also on the capability of the digital terrain model (DTM) to represent the topographic structures.

EGM2008 is commonly used to estimate the omission error (Hirt et al. 2010; Alothman et al. 2016; Ferreira et al. 2013; Godah et al. 2015; Bomfin et al. 2013; among others). But, as established in Yi and Rummel (2013), it will not show a good performance in those areas where the surface gravity data available for the development of EGM2008 were poor or fill-in data. This is the case for the southern part of Tierra del Fuego.

In this study, the omission error will be determined in three different ways: using EGM2008 at the short wavelengths, by extending the spectral content of EGM2008 with RTM data, and by computing the omission error from RTM data only.

DATA and methodology

MLL data

Fagnano Lake is located in the south-western part of Tierra del Fuego, the southernmost province of Argentina. It has an east–west extension of 100 km (Fig. 1), is 7 km wide, and has a maximum depth of 206 m (Lodolo et al. 2007). It is surrounded by mountains whose heights vary from 200 to 1000 m.

After two campaigns of GPS buoy observations, together with the information provided by three tide gauges on the lake bed (Richter et al. 2010), Del Cogliano et al. (2007) determined the MLL. This surface, which approximates an equipotential surface, was tied to the vertical datum of Tierra del Fuego through GPS/levelling points as explained in Del Cogliano et al. (2007) and Gomez et al. (2013). The accuracy of these 81 MLL observations was estimated at 5 cm RMS for one buoy position, and 2 cm RMS for the entire profile.

The uncertainty of the MLL observations depends not only on the uncertainties of the GPS buoys and tide gauge observations, but also on spatio-temporal lake level variations. The tide gauge measurements were used to reduce the instantaneous buoy observations to the mean lake level. The MLL accounts for the major driving mechanisms like the response to atmospheric forcing, seiches, water volume fluctuations, and lake tides, among others (Richter et al. 2010; Del Cogliano et al. 2007).

As noted in the paragraphs above, MLL observations are derived directly from GPS observations which were

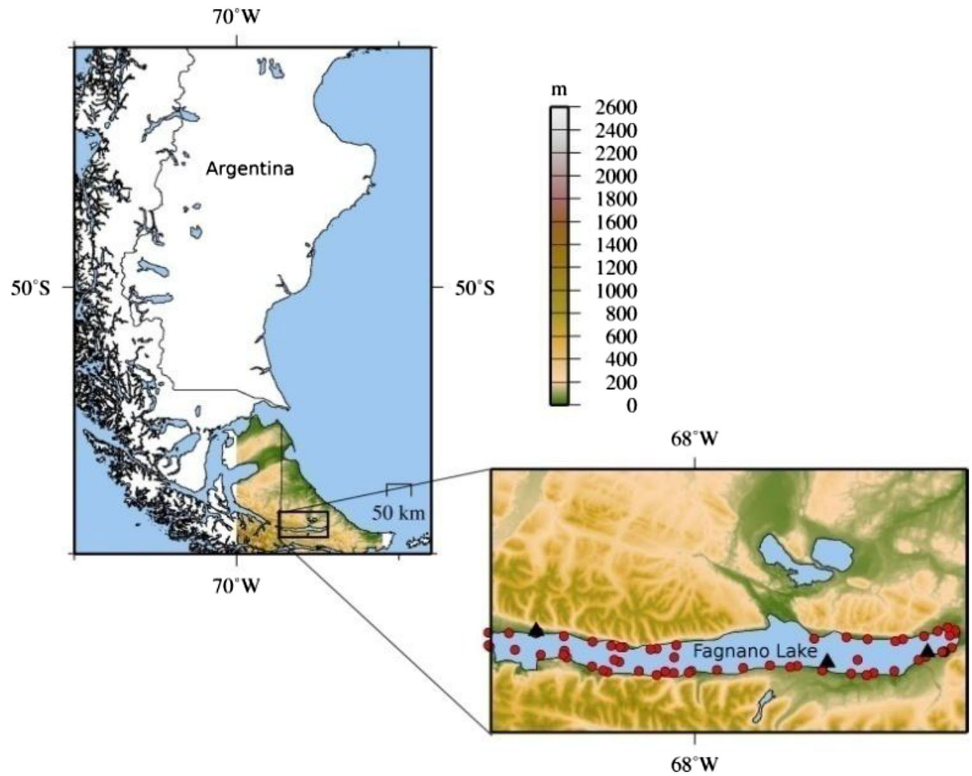


Fig. 1 The study region, located in southern Argentina. The enclosed rectangle shows the area and topography surrounding Fagnano Lake. Additionally, it shows the distribution of MLL observations (red dots) and tide gauges (black triangles). The topography surrounding the lake is represented by SRTM digital terrain model (Farr et al. 2007)

processed following the IERS Convention 2003 (Mc Carthy and Petit 2004). Therefore, they are expressed in a tide-free system (Richter, pers. comm.).

GNSS/levelling

GPS coordinates of 67 levelling marks were determined during several field campaigns in the Argentine part of Tierra del Fuego Island. The accuracy of the ellipsoidal heights is 3 cm.

Regarding the height system, all levelling data located in this part of Tierra del Fuego references the Ushuaia tide gauge. The accuracy of the levelling information (in millimetres), is in agreement with the levelling specifications of 3 mm $\sqrt{k}$, where k is the length of the levelled line in km. In the case of Tierra del Fuego, this implies an accuracy of about 1 cm, whereas in other regions it is close to mm accuracy (IGN 2016) since the levelling lines constitute closed circuits.

This information allows the determination of the geoid or quasigeoid with an accuracy of about 3 cm. In this study, no distinction was made between geoid height and height anomalies because, as established in Gomez et al. (2014), the difference between these gravity field functionals is less than 2 cm in the study region. Moreover, this difference was insignificant when compared to the discrepancies between GGMs and GNSS/levelling or MLL data in the Fagnano Lake area.

Concerning the tide system, GNSS/levelling results refer to a tide-free system.

Global geopotential models

As established by Pavlis (2010), GGMs are mathematical approximations to the external gravitational potential of an attracting body, which in this case is the Earth.

The disturbing gravitational potential, T, written in spherical harmonics, can be developed up to a certain maximum degree (Nmax) as expressed in Eq. 1 (Heiskanen and Moritz 1967; Torge 2001):

$$T(r,\lambda,\varphi) = \frac{\mathrm{GM}}{r}\sum_{n=2}^{N\max}\left(\frac{R}{r}\right)^{n}\sum_{m=0}^{n}P_n^m(\sin(\varphi))\big(\bar{C}_{nm}^{T}\cos(m\lambda) + \bar{S}_{nm}^{T}\sin(m\lambda)\big), \tag{1}$$

where GM and R are the geocentric gravitational constant and the mean equatorial radius of the Earth, respectively.

$P_n^m\sin(\varphi)$ denotes the fully normalised associated Legendre functions. $\bar{C}_{nm}^{T}$ and $\bar{S}_{nm}^{T}$ are the residual harmonic coefficients derived from the difference between the actual and normal gravity field. Finally, (r, λ, ϕ) represent the spherical geocentric coordinates of the specific point where the disturbing potential is computed.

The zero and first-degree term are assumed to be zero because it is considered that the mass of the Earth and the mass of the reference ellipsoid coincide. The implications of this assumption will be discussed later.

Table 1 provides a list of the models used in this work, and their main characteristics. At the time this work started, the selection was made because it was considered that they represented the latest combined models which involve GOCE data, thus allowing the investigation of the evolution of these GGMs in regions like the southern Andes. Although they are all combined models, they differ in the solution combination technique, as well as in the satellite and the terrestrial data included. EGM2008 (Pavlis et al. 2012) was used because it can be considered to be a classical geopotential model, and the first GGM that reached a global resolution of 9 km. It has full resolution to degree and order 2159 of the spherical harmonic expansion and provides additional coefficients to degree 2190 and order 2159 (Hirt et al. 2010; Pavlis et al. 2012). This resolution depends on the availability of accurate topographic data.

As seen in Table 1, with the exception of EGM2008, the models include GOCE data. They differ in the number of days of GOCE and GRACE data considered, as well as the altimetry and terrestrial data used.

GOCO05C is one of the latest combined models based on almost 4 years of GOCE, as well as GRACE, data and a grid of DTU13 gravity anomalies (Andersen et al. 2014) whose spatial resolution is $15' \times 15'$. It is independent of EGM2008, while EIGEN6C4 and GECO include EGM2008 information to reach their maximum degree of the spherical harmonic expansion.

GGM05C includes DTU13 terrestrial gravity anomalies in addition to the GOCE and GRACE data listed in Table 1. It combines surface gravity information with GGM05G to obtain the GGM05C solution (Ries et al. 2016).

EIGEN6C4 is a combined GGM up to degree 2190. It includes LAGEOS data, as well as 10 years of GRACE data. It also contains GOCE-SGG information. A grid of DTU10 global gravity anomalies plus EGM2008 from degree 370 onwards allows the model to reach its maximum degree.

Finally, GECO integrates the GOCE TIM R5 and EGM2008 information (Gilardoni et al. 2016).

Except GOCO05C, all of the listed GGMs are expressed in the tide-free system, as far as the permanent tide is concerned. Here, the GOCO05C model was also used in the tide-free version. The conversion can be done by means of the correction given by Smith (1998), which only affects the C20 coefficient. The ICGEM web service provides geoid heights in the three possible tide systems: tide-free, zero-tide, and mean-tide.

The analysis of GGMs can be divided into two parts: a global and a regional spectral analysis.

Global analysis

A global analysis of the behaviour of the GGMS can be done considering the degree and error degree variances of each model, which are derived from the coefficients ($\bar{C}^T_{nm}$ and $\bar{S}^T_{nm}$) in Eq. (1), and their variances ($\sigma^2_{\bar{C}_{nm}}$ and $\sigma^2_{\bar{S}_{nm}}$), respectively.

According to Rapp (1986), “the signal degree variances represent the amount of the signal contained in each degree or up to a specific degree (if computed cumulatively), while the error degree variances represent the total error power of the model at a given degree”. The latter represent the formal errors (Hirt et al. 2015).

Signal degree variances (DV) and error degree variances (EDV) per degree (n), in the geoid heights were computed based on the following formulas (Eq. (2), (3); Tsoulis and Patlakis 2013):

$$\mathrm{DV}(n) = R^2 \sum_{m=0}^{n} \left(\bar{C}^{T^2}_{nm} + \bar{S}^{T^2}_{nm} \right) \tag{2}$$

$$\mathrm{EDV}(n) = R^2 \sum_{m=0}^{n} \left(\sigma^2_{\bar{C}_{nm}} + \sigma^2_{\bar{S}_{nm}} \right) \tag{3}$$

where R is the mean Earth radius.

It is also possible to compare two GGMs using one of them as a reference. This allows the improvement evaluation in the GGM geoid heights to be compared to the geoid heights computed from the reference GGM. For the present case, EGM2008 is the reference GGM used to investigate the contribution that GOCE and new terrestrial data provide to the newest GGMs. This comparison is based on the gain determination (Eq. 4) as explained in Sneeuw (2000).

Table 1 List of the five GGMs used and their data classification according to the ICGEM web page: S (satellite gravity), G (terrestrial gravity), and A (altimetry) data

Model	Year	N_{max}	Data	References
EGM2008	2012	2190	S(Grace), G, A	Pavlis et al. (2012)
EIGEN6C4	2014	2190	S(Goce, Grace, Lageos), G, A	Förste et al. (2014)
GECO	2015	2190	S(Goce), EGM2008	Gilardoni et al. (2016)
GGM05C	2016	360	S(Grace, Goce), G, A	Ries et al. (2016)
GOCO05C	2016	720	S(Grace, Goce, SLR), G, A	Fecher et al. (2017)

$$\mathrm{Gain}(n) = \frac{\mathrm{EDV}(n)^{\mathrm{ref}}}{\mathrm{EDV}(n)^{\mathrm{new}}} \quad (4)$$

where $\mathrm{EDV}(n)^{\mathrm{ref}}$ and $\mathrm{EDV}(n)^{\mathrm{new}}$ are the error degree variances of the reference model and the new model, respectively; both computed per degree n.

Regional analysis

The regional analysis can be done through the comparisons of GNSS/levelling results with geoid heights (N) or height anomalies (ζ) computed from a GGM up to a particular expansion degree. However, GNSS/levelling height anomalies contain the full gravity spectrum, whereas the GGMs include only the gravity spectrum expanded up to its maximum degree (Nmax). To overcome this situation, Gruber et al. (2011) suggested subtracting the high-frequency components from the GNSS/levelling results before comparing them with GGMs' height anomalies or geoid undulations.

The high frequencies can be removed by means of a long wavelength GGM like EGM2008. For example, consider a GGM, different from EGM2008, with a maximum degree of expansion up to Nmax. The comparison of these GGM height anomalies with GNSS/levelling data can be made in the following way:

$$D\zeta = \zeta_{\mathrm{GNSS/levelling}} - \left(\zeta_{\mathrm{egm2008}}\big|_{(N\mathrm{max}+1)}^{2190}\right) - \zeta_{\mathrm{GGM}}\big|_{0}^{N\mathrm{max}} \quad (5)$$

Applying Eq. (5) gives GNSS/levelling results in a similar (not equal) frequency band up to Nmax, like the considered GGM. Of course, this mechanism does not remove the highest frequencies beyond degree 2159. It must be taken into account that although the EGM2008 spherical harmonics are developed up to 2190, it has full resolution up to degree and order 2159.

In Eq. (5) it is seen that EGM2008 is used to estimate the short wavelength part of the height anomalies. Thus, it is used to estimate the omission error of the GGM. In this work, the methodology proposed by Hirt et al. (2010) was applied. However, as mentioned in the introduction, EGM2008 does not include observed gravity data in the south-west of Tierra del Fuego Island. Therefore, the omission error was also determined by considering the RTM effect on height anomalies.

RTM height anomalies

Short wavelength effects can be taken into account by means of an RTM reduction (Forsberg 1984). According to Hirt (2013), this technique is capable of modelling the major part of the omission error at shorter scales than 5′.

The RTM approach implies a density reference model which has crustal density up to the height of the reference surface. A DTM representing the Earth's topography is referred to that reference surface. This leads to a residual topography which accounts for the high frequency of the gravity field spectrum if the reference surface has the same wavelength as the GGM used (Forsberg 1984; Forsberg and Tscherning 1981; Rizos and Willis 2011).

SRTM3 (Farr et al. 2007) completed with the Fagnano Lake bathymetry (Lodolo et al. 2007) and SRTM30 plus were used to represent the topography. The mean reference surface was obtained by smoothing this combined model, in agreement with the maximum degree used for each GGM. The smoothing was performed by the SELECT routine (in mode 3) of the GRAVSOFT package (Forsberg 2003) which allows the generation of a mean height grid surface from a dense DTM. For example, when working with a maximum degree of 360, the mean reference surface should have a wavelength of around 100 km (Forsberg 1984).

Although DTM2006 (Pavlis 2007) was considered, it was not employed in this study because it does not include information about the bathymetry of the lake, and it has a lower resolution than the combined DTM.

RTM effects on height anomalies were estimated by a prism-integration method where each elevation z_{RTM} represented the difference between z_{SRTM} and the mean reference surface. These differences were arranged in such a way that they could be interpreted as a grid of rectangular prisms. The gravitational potential of each prism (V^{P}) with corner coordinates $(x1, y1, z1)$ and $(x2, y2, z2)$ reads:

$$\begin{aligned} V^{\mathrm{P}}(x,y,z) = G\rho_0 \Big|\Big|\Big| & xy\ln(z+r) + yz\ln(x+r) \\ & - \frac{x^2}{2}\tan^{-1}\left(\frac{yz}{xr}\right) - \frac{y^2}{2}\tan^{-1}\left(\frac{zx}{yr}\right) \\ & - \frac{z^2}{2}\tan^{-1}\left(\frac{xy}{zr}\right)\Big|_{x_1}^{x_2}\Big|_{y_1}^{y_2}\Big|_{z_1}^{z_2} \end{aligned} \quad (6)$$

Equation (6) was proposed by Nagy et al. (2000) and it assumes prisms of constant density ρ_0, which in this case is 2.67 g cm^{-3} on land, and 1.00 g cm^{-3} on the lake.

Applying a variant of Bruns formula, the RTM effect on height anomalies can be written as (Heiskanen and Moritz 1967, p. 293):

$$\zeta_{\mathrm{RTM}} = \frac{V_{\mathrm{RTM}}}{\gamma_Q}, \quad (7)$$

where V_{RTM} is the sum of the prism gravitational contribution at the computation point represented by Eq. (6), and γ_Q is the normal gravity on point Q located on the telluroid (Heiskanen and Moritz 1967; Torge 2001, p. 257). This effect was computed by means of the TC routine (Forsberg 1984) which was modified to account for the density of the lake.

As established in Forsberg (1984), the gravitational formulas for the rectangular prisms are computationally slow and numerically unstable at large distances because they involve small differences between large numbers, which correspond to the corners of the prisms. Therefore, approximate formulas are needed at larger distances. Such formulas are based on an expansion of the prism field in spherical harmonics, which provide the simple expansion given by McMillan (1958):

$$V^{P} = G\rho\Delta x\Delta y\Delta z\left\{\frac{1}{r} + \frac{1}{24\gamma^{5}}\left[\left(2\Delta x^{2} - \Delta y^{2} - \Delta z^{2}\right)x^{2} + \left(-\Delta x^{2} + 2\Delta y^{2} - \Delta z^{2}\right)y^{2} + \left(-\Delta x^{2} - \Delta y^{2} - 2\Delta z^{2}\right)z^{2} + \frac{1}{288r^{9}}\left(\alpha x^{4} + \beta y^{4} + \ldots\right) + \ldots\right]\right\} \quad (8)$$

$\Delta x = x_2 - x_1, \Delta y = y_2 - y_1, \Delta z = z_2 - z_1$

α, β, and γ depend on the distance between the mass element and its distance to the computation point P.

In the innermost zone, around the computation point, the DTM is densified using bicubic spline interpolation. This gives place to a “finer” (a greater number) set of prisms that are used to minimise the usual large effects of the inner zone (Forsberg 1984).

Results and discussion

Global analysis

From the global spectral analysis of the square root of DV per degree, it can be seen that all GGMs retained full power almost up to their maximum (Fig. 2a). In fact, many of the GGMs did not show visible differences when inspecting the power spectra up to the maximum degree of each model. This was the case for GECO, EIGEN6C4, and EGM2008, because the first two GGMs are not independent from EGM2008, as mentioned in the previous sections. Figure 2a, shows enlarged sections at those degrees where GGM05c, GOCO05c, and the remaining models reach their maximum degree of the spherical harmonic expansion. When the EDV were considered (Fig. 2b), and EGM2008 was used as the reference GGM, the error in all of the remaining models was less than that of EGM2008 up to $n = 200$. This is because the GOCE models are more accurate than EGM2008 in the medium frequencies up to degree 200. The error increased or remained equal beyond degree 230/240, except for EIGEN6C4. The abrupt reduction of its formal error from degree 370 onwards is due to the extension of this GGM with the DTU12 global grid of gravity anomalies (Andersen et al. 2009).

EIGEN6C4 and GOCO05C showed similar behaviour in the error per degree up to approximately degree 220. According to Fig. 2b, the EDV of all models differed in the low frequencies of the spectrum, which indicates that this difference is due to the satellite data. This result is confirmed in Fig. 3. It shows the gain of the GGMs with respect to EGM2008. It can be seen that GOCE and GRACE data contributed to the long wavelengths, except for degrees higher than 240 where there was no gain with respect to EGM2008. It remained equal to 1 or less, even for GOCO05C. A different situation is seen for EIGEN6C4, which showed an improvement by a factor of 10 when compared to EGM2008 for degrees higher than 360. This is related to the gravity anomaly data mentioned above.

A comparison between EGM2008 and EGM96 (Lemoine et al. 1998) is also included to display a typical case of significant improvement. All other models, however, are evaluated with respect to EGM2008.

Regional analysis

The evaluation of the aforementioned GGMs on GNSS/levelling points, located in the Argentine part of Tierra del Fuego Island, gave the statistics shown in Table 2. The standard deviations are mostly in agreement with the accuracy published for these GGMs (see introduction section). This was an expected result because the GNSS/levelling points are located in areas with observed gravity information which has been provided to several agencies involved in global geopotential modelling. However, evaluating the GGMs using MLL data on Fagnano Lake lead to a different result (Table 3). This behaviour is explained by the presence of mountains and steep topographic gradients. In this region, the statistics were quite different, around 0.30 m in standard deviation for most of the models (EGM2008, GECO, and EIGEN6C4).

A bias correction was applied to all comparisons between observations and GGM solutions to avoid any offset caused by datum discrepancies between the local vertical datum and the geoid models (Sansò and Sideris 2013).

As established in Hirt et al. (2010) this bias correction eliminated the impact of the neglected zero and first-degree terms, and it is consistent with those studies that use standard deviation to evaluate the performance of GGMs.

Figures 4 and 5 show the RMS per degree after the evaluation of the five GGMs used for Fagnano Lake, and for the rest of the island, respectively. This computation was performed by applying Eq. (5), i.e., the high frequencies were removed using EGM2008 from degree $N\max + 1$ up to 2190, being $N\max$: 10, 20, 30… and so on, before comparing with each GGM. This was done in

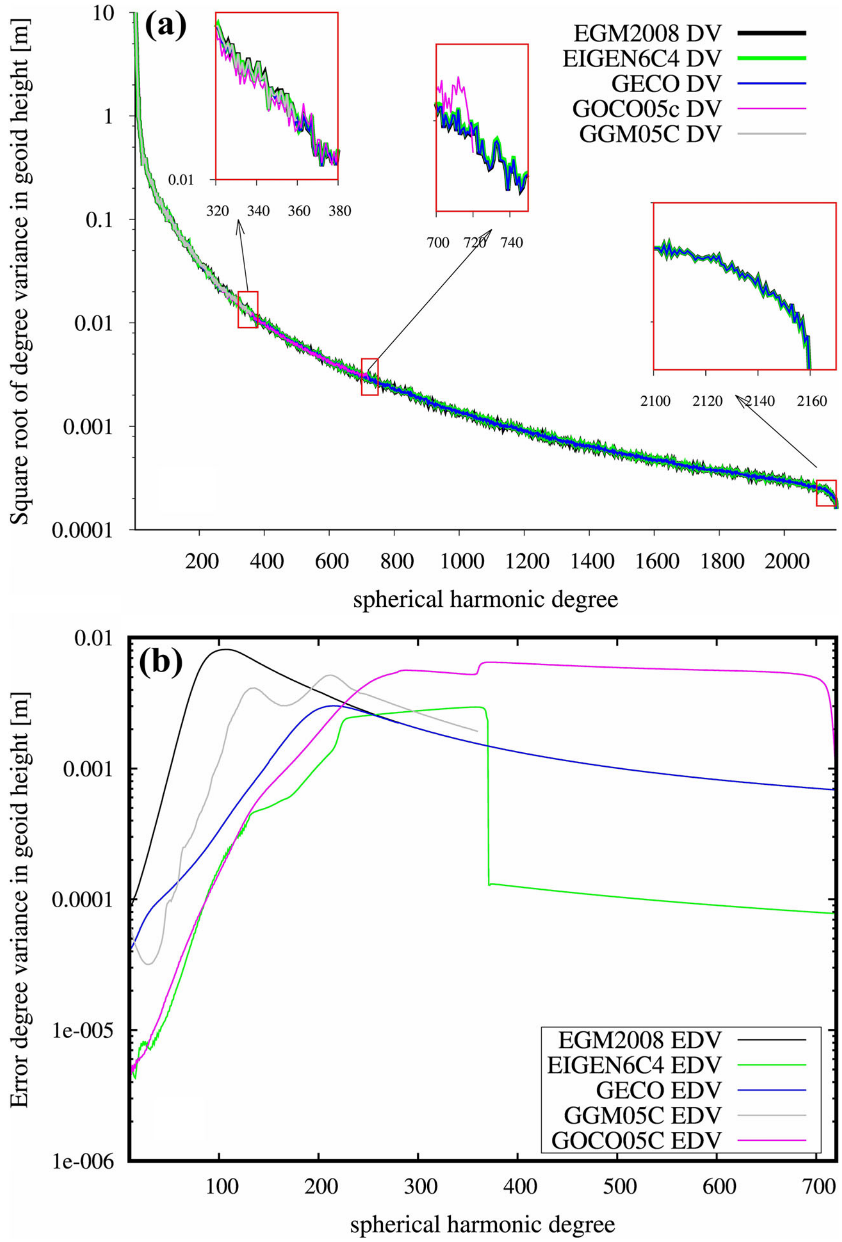


Fig. 2 **a** DV of the selected GGMs. To appreciate the differences, the inset boxes show enlarged sections at those degrees where the models reach their maximum degree of the spherical harmonic expansion; **b** EDV of the GGMs

steps of 10 degrees of spherical harmonic expansion, and MLL data were used for the comparison within the lake area (Fig. 4). The RMS increased from degree 200 to remain constant from degree 400 onwards.

In the case of GOCO05C, the RMS decreased from degree 200–720. For degree 720 it had the same accuracy as if the effect of the high frequencies had not been removed with the EGM2008 model.

This behaviour was different when evaluating the GGMs in regions covered by gravity data (Fig. 5). The difference in RMS also became evident. The removal of the high-frequency signal from GNSS/levelling data made the result better than or equal to those shown in Table 2.

According to Figs. 4 and 5, GOCO05C showed identical behaviour to the remaining models at the lowest frequencies. But major differences appeared above approximately degree 400. It was the only model that showed the same accuracy in the lake area and on land. This is confirmed by the statistics shown in Tables 2 and 3. This fact can be attributed to some improvement in the gravimetric information contained in this GGM which was not included in previous models.

Local improvement of GGMs

The aim of this section is to show the computed differences in height anomalies between MLL data and GGMs

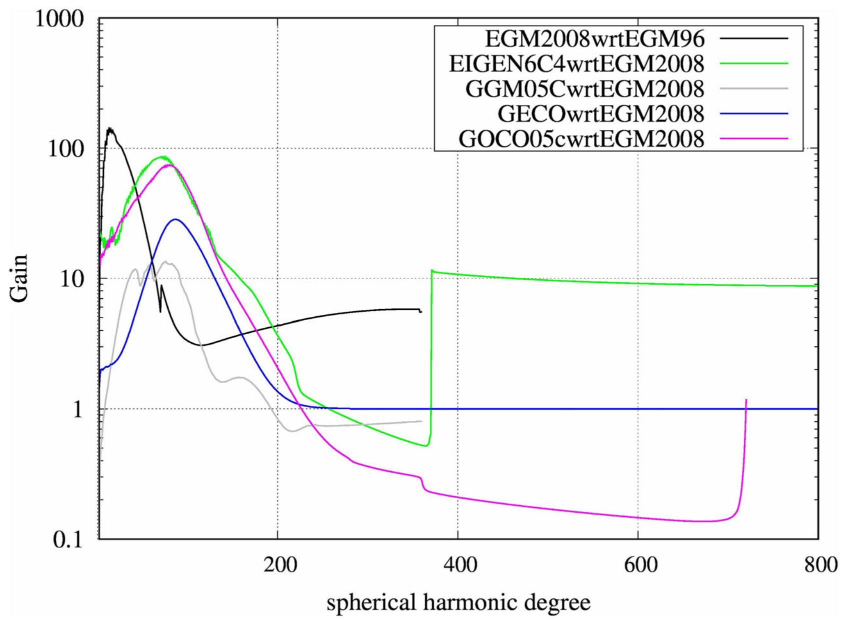


Fig. 3 Gain of GGMs w.r.t. EGM2008 per degree

Table 2 Evaluation of the GGMs selected on 67 GNSS/levelling points located in the Argentine part of Tierra del Fuego, after removing bias

Model (N_{max})	SD [m]	Max [m]	Min [m]
EGM2008(2190)	0.11	0.22	−0.25
EIGEN6C4(2190)	0.12	0.24	−0.24
GECO(2190)	0.10	0.18	−0.17
GGM05c(360)	0.16	0.41	−0.27
GOCO05C(720)	0.10	0.20	−0.26

The evaluation does not include Fagnano Lake MLL observations

Table 3 Evaluation of the GGMs on 81 MLL observations, after removing bias

Model (N_{max})	SD [m]	Max [m]	Min [m]
EGM2008(2190)	0.35	0.49	−0.8
EIGEN6C4(2190)	0.27	0.38	−0.71
GECO(2190)	0.30	0.46	−0.68
GGM05C(360)	0.15	0.34	−0.5
GOCO05C (720)	0.13	0.25	−0.38

augmented with RTM data as well as EGM2008 + RTM solutions.

Height anomalies derived from GGMs were estimated at different degrees of expansion (N_{max}):360, 720, 1080, and 2190. Table 4 shows the differences:

$$D\zeta_{MLL} = \begin{cases} \zeta_{MLL} - (\zeta_{GGMi}|_2^{N\max} + \zeta_{RTM}(N\max + 1)) \\ \zeta_{MLL} - (\zeta_{GGMi}|_2^{N\max} + \zeta_{egm2008}|_{(N\max+1)}^{2190} + \zeta_{RTM}(2160)) \end{cases} \quad (9)$$

with GGMi being each of the five GGMs up to different degree Nmax. In the case of EGM2008, just the first option of Eq. (9) was applied.

The solutions listed in Table 4 were examined, along with their gain, with respect to the EGM2008 (2190) solution. The gain shown in Table 4, was determined using a variant of Eq. (4) regarding the standard deviation (STD) of each solution, such that:

$$\text{Gain} = \frac{\text{STD}(\text{DN}_{\text{egm2008(2190)}})}{\text{STD}_{\text{GGMi}}(\text{DN}_{\text{MLL}})} \quad (10)$$

with $\text{DN}_{\text{egm2008(2190)}}$ being the difference between MLL observations and the EGM2008 (2190) solution for height anomalies.

For some of these solutions, the omission error was modelled through the RTM approach, while in other cases this result was combined with EGM2008 to complete the estimation. The starting point was degree 360, because it is the minimum degree of a combined model like GGM05C. Degree 720 corresponds to the maximum wavelength of “fill-in” data in EGM2008 for areas without observed gravity data. This “fill-in” data were generated using RTM gravity anomalies obtained from DTM2006 and a satellite-only model (Pavlis 2007).

As seen in table, there was an improvement with respect to the EGM2008 (2190) solution, especially when it was augmented with RTM data. As expected, based on the results presented in Table 4, GOCO05C (720) was the best GGM, even without any RTM improvement. According to Fecher et al. (2017), GOCE data produced a visible impact, even for higher degrees, in areas where “fill-in” data had been applied. However, the improvement was related to the

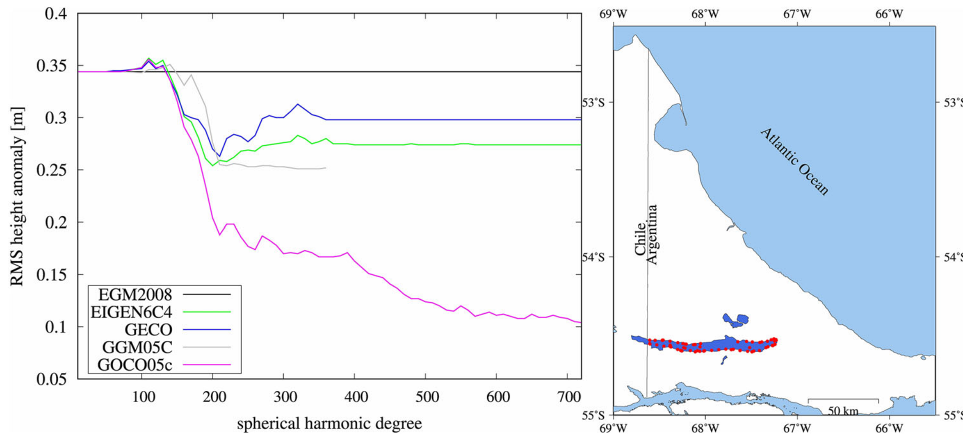


Fig. 4 RMS of the differences between geoid heights derived from GGMs and MLL observations on Fagnano Lake, after applying Eq. (5). The spatial distribution of the MLL data (red dots) is shown on the right

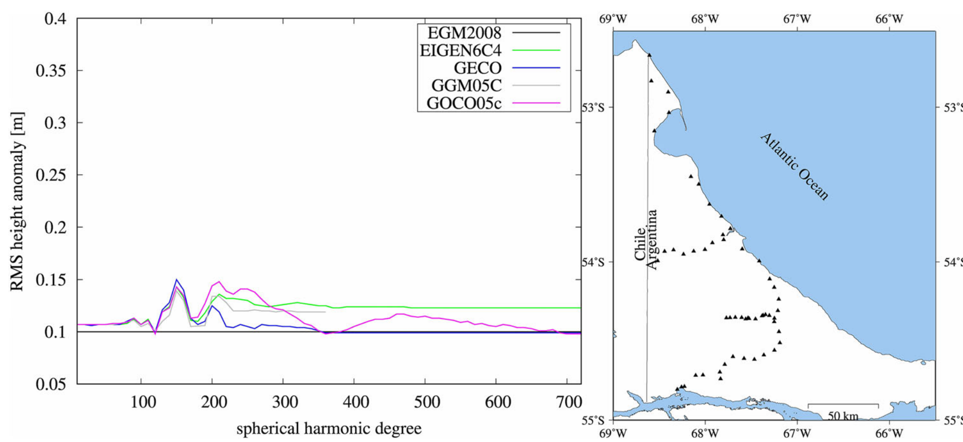


Fig. 5 RMS of the differences between GGMs and observations on GPS/levelling points over areas covered by gravity data, after applying Eq. (5). The spatial distribution of the GPS/levelling data (black triangles) is shown on the right

incorporation of new terrestrial information made by some agencies.

Differences between observed height anomalies and those derived from the GGMs up to their maximum degree are plotted in Fig. 6. To appreciate the contribution of RTM information, Fig. 7 depicts the difference between observed height anomalies and the best solution provided by each GGM after augmenting them with RTM data (according to Table 4).

Figure 6 shows that the discrepancies between GGMs and MLL data increased progressively to the west. There are two main reasons that explain this situation: the west side of the Fagnano Lake is close to the Andes Mountains, and it is characterised by steep topographic gradients. Secondly, most of the GGMs do not include observed gravity anomalies of this region as input data.

Figure 7 shows that the selected solutions produced better behaviour than EGM2008 (2190) when compared to the observations, not only in the western part of the lake but also in the east. Figure 7 also shows that a low standard deviation does not always imply a significant improvement in the western part, which is the most critical region.

Even after improvements have been made, the discrepancies in the western part of the lake were still greater than on the eastern side. This is because the observed gravity information included into the GGMs is from the north and east side of Tierra del Fuego Island (Pacino and Tocho 2009; Pavlis et al. 2012).

According to Fig. 7, GGM05C (360) and GOCO05C (720), improved by applying RTM corrections, yielded the best results. This is consistent with the values given in Table 4 when adding the omission error. These models are two of the latest combined models. They have the advantage of including GOCE data as well as new terrestrial information. As mentioned before, these two facts are responsible for the global and local improvement of GGMs.

Conclusion

The numerical results obtained at the beginning of Sect. 3 allow the conclusion to be drawn that the gain of the newest GGMs with respect to EGM2008 takes place in the lowest degrees of the spherical harmonic expansion. This situation can be explained by the inclusion of GOCE data in the most recent GGMs.

Table 4 Statistics of the differences between observations and the solutions considered, together with their gain with respect to EGM2008 (2190)

Model (N_{max})	Omission error	STD	Max.	Min.	Gain
EGM(2190)	–	0.35	0.49	−0.8	–
EGM2008(2190)	RTM(2160)	0.3	0.43	−0.73	1.17
EGM2008(1080)	–	0.39	0.51	−0.86	0.9
EGM2008(1080)	RTM(1081)	0.36	0.8	−0.55	0.97
EGM2008(720)	–	0.34	0.48	−0.8	1.03
EGM2008(720)	RTM(721)	0.32	0.47	−0.77	1.09
EGM2008(360)	–	0.2	0.31	−0.63	1.84
EGM2008(360)	RTM(361)[a]	0.2	0.3	−0.59	1.84
EIGEN6C4(2190)	–	0.27	0.38	−0.71	1.3
EIGEN6C4(2190)	RTM(2160)	0.23	0.34	−0.63	1.52
EIGEN6C4(1080)	–	0.31	0.4	−0.77	1.13
EIGEN6C4(1080)	RTM(1081)	0.28	0.36	−0.71	1.25
EIGEN6C4(1080)	EGM(2190-1081) + RTM(2160)	0.23	0.33	−0.64	1.52
EIGEN6C4(720)	–	0.31	0.39	−0.72	1.13
EIGEN6C4(720)	RTM(721)	0.28	0.37	−0.69	1.25
EIGEN6C4(720)	EGM(2190-721) + RTM(2160)	0.23	0.32	−0.64	1.52
EIGEN6C4(360)	–	0.16	0.34	-0.54	2.19
EIGEN6C4(360)	RTM(361)[a]	0.15	0.34	−0.53	2.33
EIGEN6C4(360)	EGM(2190-361) + RTM(2160)	0.24	0.34	−0.64	1.46
GECO(2190)	–	0.3	0.46	−0.68	1.17
GECO(2190)	RTM(2160)	0.16	0.41	−0.62	2.19
GECO(1080)	–	0.34	0.48	−0.74	1.03
GECO(1080)	RTM(1081)	0.31	0.43	−0.69	1.13
GECO(1080)	EGM(2190-1081) + RTM(2160)	0.25	0.4	−0.625	1.4
GECO(720)	–	0.33	0.46	−0.71	1.06
GECO(720)	RTM(721)	0.31	0.44	−0.68	1.13
GECO(720)	EGM(2190-721) + RTM(2160)	0.25	0.4	−0.625	1.4
GECO(360)	–	0.16	0.28	−0.52	2.33
GECO(360)	RTM(361)[a]	0.15	0.28	−0.51	2.33
GECO(360)	EGM(2190-361) + RTM(2160)	0.25	0.4	−0.625	1.4
GGM05c(360)	–	0.15	0.34	−0.5	2.33
GGM05c(360)	RTM(361)[a]	0.14	0.33	-0.49	2.5
GGM05c(360)	EGM(361-2190) + RTM(2160)	0.21	0.29	−0.63	1.67
GOCO05C(720)	–	0.13	0.25	−0.38	2.69
GOCO05C(720)	RTM(720)	0.11	0.23	−0.35	3.18
GOCO05C(720)	EGM(721-2190) + RTM(2160)[a]	0.1	0.19	−0.3	3.5
GOCO05C(360)	–	0.13	0.31	−0.29	2.69
GOCO05C(360)	RTM(360)	0.12	0.31	−0.28	2.92
GOCO05C(360)	EGM(360-2190) + RTM(2160)	0.17	0.3	−0.44	2.06

The gain was obtained applying Eq. (10)

[a]Indicate the best solutions, plotted in Fig. 7

It can also be appreciated that, as the GGMs evolve, their solutions are increasingly approaching the observed values. This is related to the newest gravimetric missions, better combination techniques, and new terrestrial data.

RTM height anomalies were estimated in the area of Fagnano Lake to better estimate the omission error of GGMs in that region. As seen in this work, the RTM technique has served to estimate omission errors from approximately 360 degrees onwards, but it does not make a significant contribution at higher degrees for most of the GGMs.

According to the statistics shown in Table 4 and the shape of the solutions depicted in Fig. 7, the best solution

Fig. 6 Differences in geoid height between GGMs and MLL data

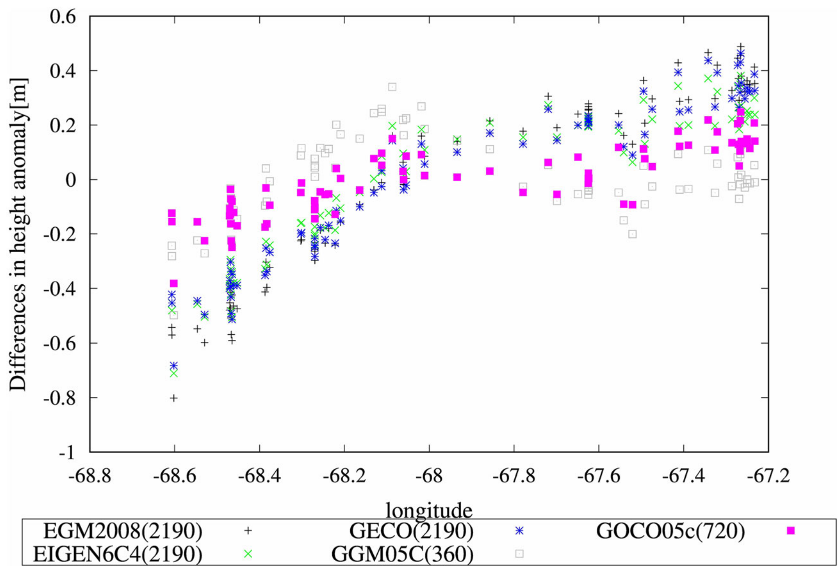
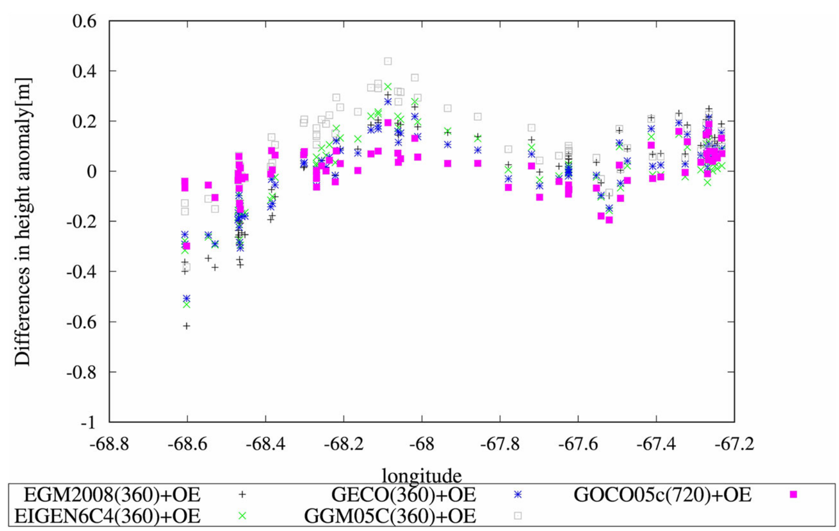


Fig. 7 Differences between the best solutions obtained with each model and MLL observations. The omission errors (OE) are indicated in Table 4 with (asterisk) for each GGM



is obtained by adding RTM height anomalies to GOCO05C.

EGM2008 does not include observed terrestrial gravity information in the western part of the Fuegian Andes, and this becomes evident in the evaluation of that model in the area of the Fagnano Lake. It has also been confirmed that the model improves when applying RTM anomalies obtained from the SRTM3 digital elevation model.

The results obtained here suggest that existing geoid models can be extended to the south-western areas of Tierra del Fuego Island by means of a better estimation of the omission error of GGMs.

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