Unstable Particles, Gauge Invariance and the Δ^{++} Resonance Parameters

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The elastic and radiative $\pi^+ p$ scattering are studied in the framework of an effective Lagrangian model for the Δ^{++} resonance and its interactions. The finite width effects of this spin-3/2 resonance are introduced in the scattering amplitudes through a complex mass scheme to respect electromagnetic gauge invariance. The resonant pole (Δ^{++}) and background contributions (ρ^0, σ, Δ, and neutron states) are separated according to the principles of the analytic S-matrix theory. The mass and width parameters of the Δ^{++} obtained from a fit to experimental data on the total cross section are in agreement with the results of a model-independent analysis based on the analytic S-matrix approach. The magnetic dipole moment determined from the radiative $\pi^+ p$ scattering is $\mu_{\Delta^{++}} = (6.14 \pm 0.51)$ nuclear magnetons.

1. INTRODUCTION

Elementary particles with spin larger than 1 have not been discovered yet. However, composite higher spin particles have been observed in nature as bound states of quarks.[1] On the other hand, the formulation of a fully consistent quantum field theory for these particles is far from being complete. Thus, the description of the dynamics of such hadronic particles is usually done in terms of an effective Lagrangian model. Such relativistic models of classical fields are built using as a guide the relevant symmetries underlying the dynamics of the specific higher spin particles. Their use for

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phenomenological purposes remains consistent as long as we restrict to a tree-level description of the amplitudes for physical processes.

Here, we consider the case where spin-3/2 particles are unstable. To be more specific, we are interested in the case of the $\Delta(1232)$ baryon resonance and in the way we introduce its finite width effects in their associated physical observables without destroying the symmetries of the effective Lagrangian (particularly, the electromagnetic gauge invariance and the invariance under contact transformations). Our goal is to provide a framework where the intrinsic properties of this particle, such as the mass, width, and magnetic dipole moment can be determined from experimental data in a consistent and well defined way. By this we mean that those properties share good physical requirements such as model independence (whenever it becomes possible), unitarity, independence upon *ad hoc* form factors, and invariance under the relevant symmetries of the interactions.

The Δ^{++} resonance has spin $J = 3/2$ and isospin $I = 3/2$. In some of the most popular Lagrangian formulations, its dynamics can be described in terms of a Rarita–Schwinger field $\psi^\mu(x)$. The dynamics of its interactions with pions, nucleons and the electromagnetic field is governed by an *effective* non-renormalizable Lagrangian.[2] Most of the problems related to the quantization of the free and the interacting theory of spin-3/2 particles[3] (see the invited talk of Prof. Sudarshan at this meeting) are absent when we use the Feynman rules to compute amplitudes only at the tree-level, as it will be the case in the present work. For the purposes of this work, such an effective Lagrangian must be able to describe the production and decay of the Δ^{++} resonance in the elastic and radiative $\pi^+ p$ scattering. As in any effective theory of the strong and electromagnetic interactions, the physical (S-matrix) amplitudes derived from our Lagrangian must be invariant under strong isospin and electromagnetic gauge transformations. Furthermore, this model must be invariant also under the so-called *contact* transformations.[4] The contact transformations are necessary to eliminate the unphysical components from the *on-shell* spin-3/2 fields. This does not prevent that the propagation of a virtual spin-3/2 resonance carries spin-1/2 components that contribute to the physical amplitude.[5] An important ingredient of our model is to use a recipe[2] to incorporate the finite width effects of the Δ^{++} resonance into the amplitude without spoiling the invariance under the above symmetries that is respected in the case of stable spin-3/2 particles.

A few more remarks are in order. The determination of the mass, width, and magnetic dipole moment (MDM) of the Δ^{++} resonance have been considered by many authors in the past (see the Particle Listings in Ref. 1). Concerning the determination of the mass and width parameters, the definitions used by authors falls into two categories: the *conventional*

and the *pole* parameters.[6] In the conventional approach, these resonance parameters are determined by applying the method of partial-wave analyses that use generalized Breit–Wigner formulae to fit the experimental data. This definition of mass and width have a strong model dependence as far as each group has its own prescription for the treatment of analiticity, the choice of background and the particular parametrization of the Breit–Wigner formula.[6] Furthermore, an unambiguous separation of resonance and background contributions in this case is hard to accomplish. In contradistinction, in the pole of the S-matrix approach the pole position is a physical property (a process- and model-independent property) of the S-matrix amplitude. Thus, the mass and width parameters of a resonance can be defined from this pole position in a more satisfactory way. It is worth to mention that the numerical values of the mass and width of the Δ^{++} defined from the pole position are significantly smaller[7] (1.7 and 15%, respectively for the mass and width) than their counterparts in the conventional approach.

On another hand, the determination of the magnetic dipole moment of a resonance is necessarily model-dependent since one is forced to specify the photon couplings to other particles. Different prescriptions to enforce gauge invariance, to incorporate the resonance character of the Δ^{++}, and to introduce other structure-dependent effects (for example, some *ad hoc* form factors) usually lead to different results for the MDM even using the same experimental data.[1] Given this diversity of theoretical methods and approximations, the PDG[1] prefers to quote an estimate for the magnetic dipole moment in the (rather wide) range $3.5 \leqslant \mu_{\Delta^{++}} \leqslant 7.5$ (in units of nuclear magnetons).

The issue of gauge invariance for processes involving unstable particles has received great attention in the last years and deserves a separate comment. This was motivated by the necessity to have a consistent definition of mass and width for the Z^0 (and W) gauge boson in view of the very precise measurements carried out at LEP. More precisely, R. Stuart has pointed out in the early nineties that the definition of the mass of the Z^0 gauge boson in the on-shell scheme was not gauge-invariant.[8] He has proposed to carry out a Laurent expansion of the full (calculated perturbatively) amplitude of $e^+e^- \to f\bar{f}$ around the pole position and separate the amplitude into resonant and background term.[8] The pole and background terms in the amplitude resulting from this expansion are separately gauge-invariant, and can provide a gauge-invariant definition for the mass and width of the Z^0 boson. This pole + background structure of the amplitude is the same as the one imposed by general arguments of the analiticity of the S-matrix that involves the production and decay of resonances.[9] Later, using the Pinch Technique, Pilaftsis and Papavassiliou[10]

were able to obtain an unstable propagator which provided a definition of mass and width parameters of a gauge-boson resonance satisfying good physical properties and, in particular, gauge invariance.

In two recent papers[11] we have extended these ideas to the sector of the baryonic Δ^{++} resonance. Using an effective Lagrangian model to describe the Δ^{++} and its interactions with the π^+, p, and the photon fields, we have been able to incorporate the finite width effects of this resonance without spoiling the symmetries of the model that are satisfied in the case of the zeroth-width approximation. In addition, the background contributions that originate from the exchange of intermediate states other than the Δ^{++} (namely, the ρ^0, σ, Δ^0, and the neutron states) are also incorporated in our effective Lagrangian model. The full amplitude obtained in our approach has the pole plus background structure dictated by the analytic S-matrix theory.[9] Each one of these terms in the amplitudes are separately gauge-invariant and the insertion of *ad hoc* form factors to restore gauge invariance is not necessary in our case.

In this talk we summarize the main aspects of our analysis. We emphasize from our results the model-independent aspects of the mass, width and magnetic dipole moment parameters that follows from our separation of pole and background contributions. In a first step, we fix the mass, the width and the strong coupling of the Δ^{++} from the elastic $\pi^+ p$ scattering. Then we obtain the Δ^{++} MDM from the radiative $\pi^+ p$ scattering observables. It is interesting to note that the elastic scattering requires the contribution of the scalar σ meson in the t-channel to get an improved fit of the data. The details of the different calculations and input data can be found in Ref. 11.

2. THE EFFECTIVE LAGRANGIAN

In this section we provide the pieces of the Lagrangian for the Rarita–Schwinger field $\psi^\mu(x)$ that are relevant to describe the Δ^{++} contributions to the elastic ($\pi^+ p \rightarrow \pi^+ p$) and radiative ($\pi^+ p \rightarrow \pi^+ p\gamma$) processes of our interest. The interaction Lagrangians for ρ^0, σ mesons and the neutron intermediate states that contribute to the background amplitude are well known and can be found for example in Ref. 11.

The Lagrangian that describes the Δ^{++} and its interactions with the pion (ϕ), proton (ψ), and photon (A_μ) fields is given by:

$$\mathscr{L}_\Delta = \mathscr{L}_0 + \mathscr{L}_{\Delta\pi p} + \mathscr{L}_{\Delta\Delta\gamma} + \mathscr{L}_{\Delta\pi p\gamma}. \tag{1}$$

The different pieces in this Lagrangian have explicitly forms:

$$\mathscr{L}_0 = \bar{\psi}^\mu \Lambda_{\mu\alpha}(A)\, G^{\alpha\beta} \Lambda_{\beta\nu}(A)\, \psi^\nu, \tag{2}$$

$$\mathscr{L}_{\Delta\pi p} = \left(\frac{f_{\Delta N\pi}}{m_\pi}\right) \bar{\psi}^\mu \Lambda_{\mu\nu}(A)\, \psi\, \partial^\nu \pi + \text{h.c.}, \tag{3}$$

$$\mathscr{L}_{\Delta\Delta\gamma} = -2e\bar{\psi}_\nu \Lambda^{\nu\nu'}(A)\, \Gamma_{\nu'\mu'\alpha} \Lambda^{\mu'\mu}(A)\, \psi_\nu A^\alpha, \tag{4}$$

$$\mathscr{L}_{\Delta\pi p\gamma} = \left(\frac{e f_{\Delta N\pi}}{m_\pi}\right) \bar{\psi}^\mu \Lambda_{\mu\nu}(A)\, \psi\, \pi A^\nu + \text{h.c.} \tag{5}$$

The rank two tensors introduced in the above Lagrangians are defined as follows:

$$G^{\alpha\beta} \equiv g^{\alpha\beta}(i\slashed{\partial} - M) + \frac{i}{3}(\gamma^\alpha \slashed{\partial} \gamma^\beta - \gamma^\alpha \partial^\beta - \partial^\alpha \gamma^\beta) + \frac{1}{3} M \gamma^\alpha \gamma^\beta, \tag{6}$$

$$\Lambda_{\mu\nu} \equiv g_{\mu\nu} + \frac{1}{2}(1+3A)\, \gamma_\mu \gamma_\nu. \tag{7}$$

We have defined the electromagnetic vertex of the Δ^{++} following Ref. 2:

$$\Gamma_{\alpha\beta\rho} = \left(\gamma_\rho - \frac{i\kappa_\Delta}{2M}\, \sigma_{\rho\sigma} k^\sigma\right) g_{\alpha\beta} - \frac{1}{3}(\gamma_\rho \gamma_\alpha \gamma_\beta + \gamma_\alpha g_{\beta\rho} - \gamma_\beta g_{\alpha\rho}), \tag{8}$$

where the Δ^{++} MDM is given by

$$\mu_{\Delta^{++}} = 2(1+\kappa_\Delta)\, \frac{e}{2M}, \tag{9}$$

and κ_Δ is the *anomalous* part of the magnetic dipole moment. In the above equations, m_π and M denote the pion and Δ^{++} masses while $f_{\Delta N\pi}$ is the (strong) coupling constant of the $\Delta N\pi$ vertex. The isospin-invariant version of the above Lagrangian can be found in Ref. 11. The Feynman rules associated to these Lagrangian can be found in Ref. 2.

The arbitrary parameters A that appears in the tensor $\Lambda_{\mu\nu}(A)$ is associated to the contact transformations acting upon the Rarita–Schwinger field ($a \neq -1/4$):

$$\psi^\mu \to \psi^\mu + a\gamma^\mu \gamma_\alpha \psi^\alpha, \qquad A \to A' = \frac{A - 2a}{1 + 4a}. \tag{10}$$

One can easily prove that the above (free and interacting) Lagrangian remains invariant under these contact transformations. As is was proven explicitly for the case of elastic and radiative $\pi^+ p$ scattering,[2] the S-matrix

amplitudes for these processes are *independent* of the arbitrary parameter A as it should be. Furthermore, the following Ward identity between the propagator $P^{\mu\nu}(p)$ (see Ref. 2) and the electromagnetic vertex of the Δ^{++} (see Eq. (8))

$$P^{\mu\alpha}(P')\, \Gamma_{\alpha\beta\rho} k^\rho P^{\beta\nu}(P) = P^{\mu\nu}(P) - P^{\mu\nu}(P'), \qquad (11)$$

assures that the S-matrix amplitude of the radiative $\pi^+ p$ process is also gauge-invariant.

   In summary, the model for the Δ^{++} and its interactions with other particles described in this section give rise to S-matrix amplitudes which are gauge-invariant and satisfy invariance under contact transformations. This conclusion holds as far as we consider the Δ as an stable particle. Introducing the decay width naively in the denominator of the propagator destroys gauge invariance. In the following section we discuss a mechanism to introduce consistently the finite width effects.

3. RECIPE FOR UNSTABLE PARTICLES

   Consider a radiative processes that is dominated by the production of a resonance in the *s*-channel. Using a propagator of an unstable as it is obtained from Dyson summation of bubble graphs and the on-shell renormalization scheme leads to an amplitude that is not invariant under electromagnetic gauge transformations. The radiative amplitude can be rendered gauge invariant if we replace $m_0^2 \to m^2 - im\Gamma$ in all the Feynman rules of the model, where m_0 is the bare mass of the particle and m (Γ) is its physical mass (width). This *complex mass* recipe was proposed as a solution to recover electromagnetic gauge invariance of the amplitude of resonant processes in Ref. 12.

   To illustrate the origin of this recipe, let us consider a resonant scalar particle as a simple example. As is well known, the self-interactions of this particle during his propagation transforms its bare propagator

$$D_0(q^2) = \frac{i}{q^2 - m_0^2}$$

into the renormalized propagator

$$D(q^2) = \frac{i}{q^2 - m^2 - iZ\,\mathrm{Im}\,\Pi(q^2)}, \qquad (12)$$

if we use the renormalization conditions $m_0^2 = m^2 - \mathrm{Re}\,\Pi(m^2)$, $Z^{-1} = 1 - \mathrm{Re}\,\Pi'(m^2)$. In this definition, m becomes the renormalized mass.

The unitary condition of the S-matrix amplitude allows to identify $Z\,\mathrm{Im}\,\Pi(q^2) = -\sqrt{q^2}\,\Gamma(q^2)$, where $\Gamma(q^2)$ is the decay width of the scalar particle with (virtual) mass q^2.

Let us consider now this renormalized propagator in a physical process. One of the simplest radiative process is the scattering reaction $\pi^+(p)\,\eta(q) \to \pi^+(p')\,\eta(q')\,\gamma(k,\epsilon)$ which we assume to be dominated by the production of the charged scalar resonance a_0^+ in the s-channel (letters within parenthesis denote the four-momenta and ϵ the photon polarization vector). There are three resonant contributions corresponding to the photon emitted from the external pion lines and from the internal a_0^+ propagator line. The transition amplitude is given by

$$\mathcal{M} = eg^2 \left\{ -\frac{p.\epsilon}{p.k}\,D(Q') + \frac{p'.\epsilon}{p'.k}\,D(Q) - iD(Q)\,D(Q')(Q+Q').\epsilon \right\}. \quad (13)$$

We have introduced the variables $Q = p+q$, $Q' = p'+q'$ ($Q = Q'+k$) which denote the four-momenta of the intermediate a_0^+ resonance, and $D(Q_i)$ denote its resonant propagator as given in Eq. (12). The factor g denotes the coupling constant for the $a_0\eta\pi$ vertex.

We can easily check that the above amplitude is not gauge-invariant, namely that $\mathcal{M}$ does not vanish when ϵ is replaced by k. Gauge invariance is not satisfied due to the presence of the (energy-dependent) imaginary part of the propagator. Gauge invariance can be restored in different forms, introducing in this way an ambiguity in the amplitude. One can for instance include form factors in the strong vertices or additional contributions to the amplitude in an *ad hoc* way. A second possibility is to include the one loop corrections to the electromagnetic vertex of the a_0^+ meson in order to satisfy a Ward identity at the one-loop level. A third option consists in using a complex mass scheme as proposed in Ref. 12.

We consider here the complex mass scheme since it provides the simplest solution. For the illustrative example under consideration, let us consider only the absorptive corrections to the propagator. If we assume a renormalized mass for the a_0 from the beginning, we can write the absorptive part of the self-energy correction as follows:

$$-\mathrm{Im}\,\Pi(s) = \sqrt{s}\,\Gamma_{a_0}(s) = \frac{g^2}{16\pi s}\left\{(s-(\mu+\mu')^2)(s-(\mu-\mu')^2)\right\}^{1/2}, \quad (14)$$

where μ (μ') denotes the mass of the η (π^+) meson running in the loop correction. In the limit of *massless* particles in the loop we can check that:

$$-\mathrm{Im}\,\Pi(s) \to \frac{g^2}{16\pi} = M\Gamma, \quad (15)$$

where Γ is the decay width of the a_0^+ meson and M its mass. Thus, the resonant propagator becomes:

$$D_{a_0}(s) \to \bar{D}_{a_0}(s) = \frac{i}{s - M^2 + iM\Gamma}. \tag{16}$$

This propagator can be obtained from the bare propagator if we simply replace the bare mass by the pole position $M^2 - iM\Gamma$, namely if we adopt the complex mass scheme. Owing to the identity:

$$\bar{D}_{a_0}(Q') \, \bar{D}_{a_0}(Q) = \frac{i}{(Q + Q') . k} \{\bar{D}_{a_0}(Q') - \bar{D}_{a_0}(Q)\}, \tag{17}$$

we can check that using the resonant propagator (16) in the limit of massless loop corrections, the amplitude Eq. (13) becomes gauge invariant.

This simple example illustrates that the complex mass scheme and the absorptive one loop corrections to the electromagnetic vertex and the propagator (in the limit of massless particles running in loop corrections[13]) are equivalent methods to restore gauge invariance. Although this approximation (massless particles in loop corrections) can hardly be justified in the case of the Δ^{++} resonant propagator (because the proton in the loop is not massless in the chiral limit), the complex mass scheme provides the simplest solution to the gauge invariance problem for resonant amplitudes and it will be adopted here for our calculations.

4. FITTING EXPERIMENTAL DATA

Just to clarify our procedure, we repeat here the main steps of our analysis. First, we use the experimental data on the total cross section of $\pi^+ p$ elastic scattering to fix some relevant free parameters (mass, decay width and strong coupling of the Δ) of the model. Then, we use the data on radiative $\pi^+ p$ scattering to fix the MDM of the Δ^{++} which remains as the only free parameter in this reaction. The details of the fit procedure and further additional tests of the model can be found in Ref. 11. Here we focus on the discussion of the relevant features of the model and the results of the fit.

4.1. Elastic $\pi^+ p$ Scattering

The model contributions to the $\pi^+ p \to \pi^+ p$ scattering includes the Δ^{++} resonance (s-channel), the ρ and σ mesons (t-channel) and the Δ^0 and

neutron states (crossed-channel) contributions. There are five Feynman diagrams corresponding to these contributions which can be found in Ref. 11. The experimental data for the total cross section is chosen to lie in the resonance region, which corresponds to pion kinetic energies $T_{\text{lab}} = 75 \sim 300$ in the lab system.[14] In this kinematical region, the elastic scattering is dominated by the production of the Δ^{++} resonance and all other terms can be considered as small background contributions. This is indeed the case, as it can be checked from Fig. 1.

The parameters entering the background contributions (except the Δ^0 mass and the couplings of the scalar meson) are taken from other low energy processes (see Ref. 11). Their precise values are not of critical importance as far as they contribute as a small term to the amplitude. Therefore, the only free parameters of the model are the mass (M_Δ), width (Γ_Δ), and $\Delta N\pi$ ($f_{\Delta N\pi}$) coupling of the Δ and the effective coupling of the scalar meson ($g_\sigma = g_{\sigma\pi\pi}\, g_{\sigma NN}$).

In order to assess the influence of the different background terms, we have performed several fits to the total cross section by adding successively the different background contributions. For the mass of the scalar σ meson
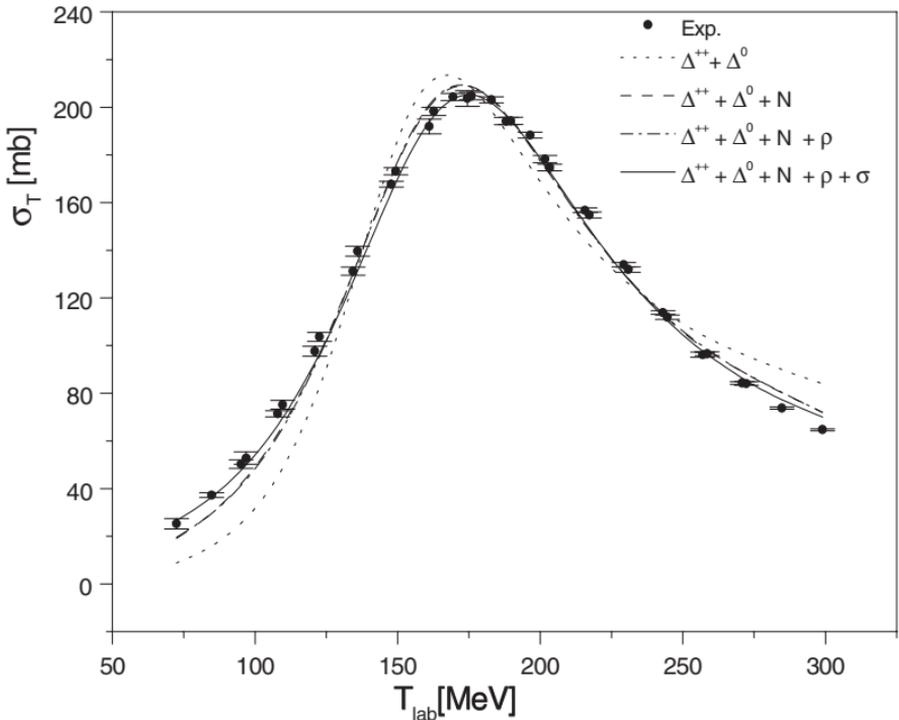
Fig. 1. Total cross section of elastic $\pi^+ p$ scattering: comparison of model and experimental data.

Table I. Fit Results to the Total Cross Section of $\pi^+ p$ Elastic Scattering

Intermediate state	$f^2_{\Delta N\pi}/4\pi$	m_Δ (MeV)	Γ_Δ (MeV)	$g_\sigma/4\pi$	χ^2/dof
$\Delta^{++,\,0}$	0.281 ± 0.001	1201.7 ± 0.2	69.8 ± 0.2	–	121.1
$\Delta^{++,\,0}, N$	0.331 ± 0.003	1208.6 ± 0.2	87.5 ± 0.3	–	17.6
$\Delta^{++,\,0}, N, \rho$	0.327 ± 0.001	1207.4 ± 0.2	85.6 ± 0.3	–	15.6
$\Delta^{++,\,0}, N, \rho, \sigma$	0.317 ± 0.003	1211.2 ± 0.4	88.2 ± 0.4	1.50 ± 0.12	10.5
Unitarity	0.317	1211.7	92.2	1.50	9.8
Pole position		1212.2 ± 0.3	97.1 ± 0.4		

we have chosen $m_\sigma = 650$ MeV. We have allowed a wide variation of this mass, namely $\Delta m_\sigma = \pm 200$ MeV, and have found that it is correlated mainly with the value of g_σ, while the other parameters are not affected in an important way. The results of the fit are shown in Table I and in Fig. 1.

A few comments are in order:

(i) The χ^2/dof drops from 121 to 10 in going from the top to the bottom of Table I, which indicates the necessity of including in the fit some degrees of freedom other than the Δ resonance. The large contributions to the χ^2/dof come from the data points for the highest pion energies considered in the fit (see Fig. 1). In Fig. 1 we can observe the best results obtained for each case indicated in Table I. Although the χ^2/dof is not indicative of a very good fit, we can expect that such a fit can be improved by considering effects of rescattering and other background terms excluded from the simple pole approximation implicit in our model.

(ii) Since only the amplitude involving the Δ^{++} resonance has an imaginary part, it can be easily checked that the complete amplitude does not satisfy unitarity. We can force our result to satisfy unitarity by adding a softly energy-dependent term to the amplitude. The presence of additional terms in the amplitude can be justified on the basis that we have kept only the pole term in our amplitude for the Δ^{++} contribution.[11] The result obtained from the fit when we impose unitarity is shown in the fifth row of Table I. Namely, unitary only shifts the value of the decay width in the right direction to match the model-independent result[7] shown in the last row of Table I.

(iii) The row denoted as "pole position" in Table I contains the results of the fit obtained in a model-independent analysis of the same data for the cross section.[7] The close agreement observed

in Table I between our model-dependent results and the model-independent analysis of Ref. 7 indicates that our model describes very well not only the resonant but also the background contributions in the amplitude.

(iv) Once the relevant parameters of the model are fixed from the total cross section, we can *predict* the differential cross section $d\sigma/d\Omega$ for pions emitted at an angle θ with respect to the incident pion beam. Our model is able to reproduce two sets of data corresponding to kinetic energies of incident pions at $T_{lab} = 263$ and 291 MeV.[11] The test of the model at these energies is important because the data used to extract the magnetic dipole moment of the Δ^{++} correspond to kinetic energies close to those values (see next section).

These important remarks indicates that our model is well suited to describe the dynamics of the $\pi^+ p$ reactions in the Δ^{++} resonance region and provides good confidence to apply it in the description of other reactions. In the next section, we use it to extract the MDM of this resonance from the data on radiative $\pi^+ p$ scattering.

4.2. Radiative $\pi^+ p$ Scattering

Once we have fixed the mass and width of the Δ and other relevant couplings of the model, it remains only one parameter to describe the radiative $\pi^+ p$ scattering: the magnetic dipole moment of Δ^{++}, namely $\mu_{\Delta^{++}}$ or κ_Δ. In this section, we fit this parameter from experimental data on $\pi^+ p \rightarrow \pi^+ p \gamma$. From the 35 Feynman diagrams (see Ref. 11) that contribute to this process in our model, seven correspond to photons emitted from process involving the Δ^{++} intermediate states and 28 are associated with the ρ^0, σ, and Δ^0, n intermediate states.

The physical observable of our interest is the five-fold differential cross section $d\sigma/d\omega_\gamma \, d\Omega_\gamma \, d\Omega_\pi$ of the radiative $\pi^+ p$ scattering. In this observable, ω_γ is the photon energy, $d\Omega_\gamma$ is the element of solid angle where photons are emitted with respect to final pions, and $d\Omega_\pi$ is the solid angle for final state pions measured with respect to the direction of incident pions. We use the data corresponding to incident pions of energies $T_{lab} = 269$ and 298 MeV.[15] As we have pointed out in the previous section, our model is still good to describe the data at those energies. The kinematical range of photon energies is $0 \leqslant \omega_\gamma \leqslant 100$ MeV, where we can expect, in a conservative way, that the soft-photon approximation is valid and that other structure dependent terms or higher electromagnetic multipoles contributions are small. In addition, we consider a few set of photon angular configurations

Table II. Anomalous Δ^{++} Magnetic Dipole Moment Extracted from Radiative π^+p Scattering

T_{lab} (MeV)	Geometry	θ_γ	ϕ_γ	κ_Δ	χ^2/dof
269	G7	120°	0°	3.27 ± 0.76	1.99
	G4	140°	0°	3.01 ± 0.67	2.48
	G1	160°	0°	2.74 ± 0.87	1.73
298	G7	120°	0°	3.10 ± 0.87	2.68
	G4	140°	0°	2.90 ± 0.75	4.75
	G1	160°	0°	2.61 ± 1.00	1.47

where the differential cross section is more sensitive to the effect of the Δ^{++} MDM (see Ref. 11).

The results of the fits for the most sensitive observables are shown in Table II (the definition of the "anomalous" part κ_Δ of the MDM was given in Eq. (9)).

Again, a few remarks are worth to be mentioned:

  (i)   In Fig. 2 we show the best fits for a few samples of the differential cross section as a function of the photon energies for $T_{\text{lab}} = 269$ MeV. Just to show the sensitivity of the chosen configurations (G1, G4, and G7) to the effect of the magnetic dipole moment, in Fig. 2 we compare the best fits of Table II (solid lines) with the curves corresponding to a reference value $\kappa_\Delta = 1$ (dashed curves).

 (ii)   Other (less sensitive to κ_Δ) measured angular configurations were also considered in the analysis. The description of data is very good as it can be checked in Ref. 11.

(iii)   The set of values determined for κ_Δ (see Table II) is remarkable consistent. This allows to quote a meaningful weighted average from the six values of κ_Δ shown in Table II. We obtain:

$$\mu_{\Delta^{++}} = 2(1 + \kappa_\Delta)\frac{e}{2m_\Delta} = (6.14 \pm 0.51)\frac{e}{2m_p}. \qquad (18)$$

Note that the last numerical value is given in units of nuclear magnetons.

5. REMARKS AND CONCLUSIONS

The contributions of the Δ^{++} resonance to the elastic and radiative π^+p scattering is revisited in the light of a consistent effective Lagrangian model
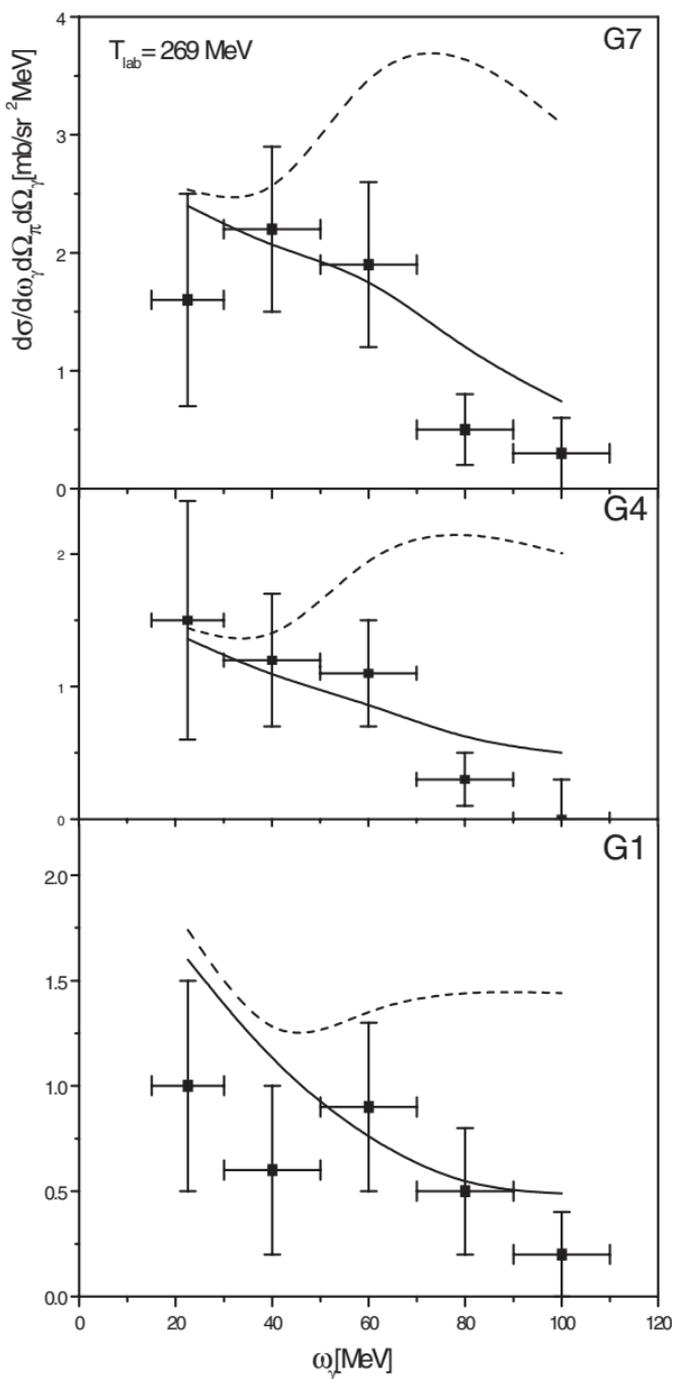
Fig. 2. Differential cross section of radiative $\pi^+ p$ scattering and best fits results for $T_{lab} = 269$ MeV and three angular configurations of photon energies.

for the spin-3/2 *unstable* particle and its interactions. Our proposal respects two very important symmetries of a theory of the spin-3/2 particles: the invariance under *contact* and electromagnetic gauge transformations. We have shown that introducing the finite width effects of the resonance through a complex mass scheme, namely replacing $M^2 \rightarrow M^2 - iM\Gamma$ in all the Feynman rules that involve the spin-3/2 particle, do not spoil these symmetries of the effective theory. Such a recipe is well motivated by recent studies concerning the search a proper (gauge-invariant) definition of the mass of an unstable gauge-boson in the framework of perturbative gauge theories.

We have performed a phenomenological analysis of this effective Lagrangian to test its viability as an acceptable model for the low energy $\pi^+ p$ scattering processes. Our approach is closely related to the one of the analytic S-matrix; namely, we try to give a physical meaning to the mass and width of the resonance from an explicit separation of resonant and background contributions in the S-matrix amplitude for the elastic scattering. By introducing the complex mass scheme in the propagator of the resonance we are able to isolate the pole contributions in a simple and clean way. The background contributions are given in our model by the exchange of the ρ^0, σ, Δ^0, and neutron intermediate states. In the case of elastic $\pi^+ p$ scattering, we have found that the mass and width of the Δ^{++} resonance are in excellent agreement (within the approximations inherent to our model) with the values obtained in the framework of the model-independent analytic S-matrix approach (see Table I). The description of the elastic scattering data for the total cross section is very good in a wide region considered around the resonance peak. The differential cross section of elastic scattering is predicted to be in good agreement with a set of data for pion kinetic energies to the right side of the resonance peak.

We have considered also the radiative $\pi^+ p$ scattering in view of extracting a value of the Δ^{++} magnetic dipole moment from the experimental data. Electromagnetic gauge invariance is fulfilled for the resonance contributions to the amplitude owing to a simple Ward identity that is satisfied between the propagators and the electromagnetic vertex of the Δ^{++} in the complex mass scheme. Our model provides a very simple solution to the gauge invariance problem for the resonance contributions to the radiative amplitude in the presence of finite width effects. Within our approach we do not need to introduce *ad hoc* form factors or additional contributions to obtain a gauge-invariant amplitude.

Using the most sensitive set of data for the five-fold differential cross section of the radiative $\pi^+ p$ scattering we are able to give a good fit with only one free parameter: the Δ^{++} magnetic dipole moment. The results for the Δ^{++} MDM are described in Sec. 4.2 and can be found in Table II and

Eq. (18). Our determination of the MDM are in good agreement with recent theoretical calculations that incorporate the QCD corrections in a chiral bag model[16] and with the predictions of a phenomenological quark model[17] that includes the non-static effects of pion exchange and orbital excitation. Our determination of the MDM is, however, a bit larger that some calculations based on the $SU(6)$ spin-flavor symmetry.[18]

In summary, we have shown that the data on the elastic and radiative $\pi^+ p$ scattering near the Δ^{++} resonance region can be well described in the framework of an effective Lagrangian model for this spin-3/2 particle. This model is free from the very well known inconsistencies present in the quantum field theory formulations of spin-3/2 particles as far as we use the model only at the tree-level. The calculations of the scattering amplitudes fully exploit the model-independent separation of the amplitude into the resonant and background contributions advocated by the analytic S-matrix theory. This allows us to give a physical meaning to the mass and width values extracted for the Δ^{++} resonance and, by extension, to its the magnetic dipole moment parameter.

ACKNOWLEDGMENTS

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