Nuclear-Matter Calculation with a Rank-One Separable Potential.

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It has been recently proposed a family of rank-one separable potentials [1] which fits the nucleon-nucleon scattering data up to 460 MeV (lab).

These potentials were tested in different calculations of two- and three-nucleon parameters. In particular we mention the fact they reproduce fairly well the off-shell behaviour of the two-nucleon system. As a consequence of these results it appears interesting to go further with the study of this new set of potentials. It is for this reason that we have investigated their behaviour in connection with nuclear matter.

It is well known that the binding energy per particle in infinite nuclear matter to lowest order in the Brueckner-Goldstone [2] theory is given by

$$\frac{E}{A} = \sum_{n \leqslant F} \left\{ \langle n|T|n\rangle + \frac{1}{2} \sum_{m \leqslant F} \langle nm|G|nm - mn\rangle \right\}, \tag{1}$$

where the sum goes over all occupied plane-wave states, T is the kinetic-energy operator and G is the reaction matrix. In terms of the relative and c.m. momenta, the G-matrix is a solution of the Bethe-Goldstone equation

$$\langle \boldsymbol{k}|G_{\boldsymbol{K}}(w)|\boldsymbol{k}'\rangle = \langle \boldsymbol{k}|V|\boldsymbol{k}'\rangle - \int \frac{\langle \boldsymbol{k}|V|\boldsymbol{p}\rangle Q(\boldsymbol{K}, \boldsymbol{p}) \langle \boldsymbol{p}|G_{\boldsymbol{K}}(w)|\boldsymbol{k}'\rangle}{e(\boldsymbol{K}, \boldsymbol{p}, w)} \mathrm{d}^3 p, \tag{2}$$

here $Q(\boldsymbol{K}, \boldsymbol{p})$ stands for the Pauli operator and the energy denominator is

$$e(\boldsymbol{K}, \boldsymbol{p}, w) = \varepsilon\left(\boldsymbol{p} + \frac{\boldsymbol{K}}{2}\right) + \varepsilon\left(\boldsymbol{p} - \frac{\boldsymbol{K}}{2}\right) - w, \tag{3}$$

where ε is the single-particle energy and w the starting energy. For the single-particle

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(2) B. DAY: *Rev. Mod. Phys.*, **39**, 719 (1967).

energy we use the following prescription:

$$(4)\qquad \begin{cases} \varepsilon_m = \dfrac{\hbar^2}{2M} k_m^2 + U(k_m), & k_m \leqslant k_F, \\ \varepsilon_a = \dfrac{\hbar^2}{2M} k_a^2, & k_a > k_F \end{cases}$$

with

$$(5)\qquad U(k_m) = \sum_{n \leqslant F} \langle mn | G_{\boldsymbol{K}}(w = \varepsilon_m + \varepsilon_n) | mn - nm \rangle .$$

In solving the Bethe-Goldstone equation we made the usual approximations: i) angle-averaged Pauli operator, ii) averaged c.m. momentum $\bar{K}$ and iii) single-particle energies of occupied states (*i.e.* $k_m \leqslant k_F$) by the reference spectrum, *i.e.*

$$\varepsilon_m = A + \frac{\hbar^2}{2M^*} k_m^2 ,$$

where A and M^* are adjusted by least squares to the spectrum obtained from eqs. (4) and (5).

Due to the fact that the present potentials are diagonal in the eigenchannel representation, the corresponding G-matrix is also diagonal in the present approximation and can be evaluated in a simple way.

The final expression for the binding energy per particle is

$$\frac{E}{A} = \frac{3}{10}\frac{\hbar^2 k_F^2}{M} + \frac{4\hbar^2}{\pi M} \sum_{J,S,T,\alpha} (2J+1)(2T+1)\sigma_\alpha^{JST} \int_0^{k_F} \frac{\left(1 - \frac{3}{2}\frac{k}{k_F} + \frac{k^3}{2k_F^3}\right)[k \cdot g_\alpha^{JST}(k)]^2 \, \mathrm{d}k}{1 + \frac{2}{\pi}\sigma_\alpha^{JST} \int_0^\infty \frac{[x g_\alpha^{JST}(x)]^2 \bar{Q}(\bar{K}, x)}{x^2 + \gamma^2} \, \mathrm{d}x} ,$$

where α labels the eigenchannel ($\alpha = 0, 1, 2$ in our case) and

$$\gamma^2 = -\frac{M}{M^*} k^2 - \frac{2MA}{\hbar^2} + \frac{\bar{K}^2}{4}\left(1 - \frac{M}{M^*}\right) .$$

In a similar way one obtains for the single-particle potential

$$U(k_m) = \frac{8\hbar^2}{\pi M} \sum_{J,S,T,\alpha} (2J+1)(2T+1) \cdot \sigma_\alpha^{JST} \cdot$$
$$\cdot \left\{ \int_0^{k_F - k_m} \frac{[k \cdot g_\alpha^{JST}(k)]^2 \, \mathrm{d}k}{1 + \frac{2}{\pi}\sigma_\alpha^{JST} \int_0^\infty \frac{[x \cdot g_\alpha^{JTS}(x)]^2 \bar{Q}(\bar{K}, x)}{x^2 + \gamma^2} \, \mathrm{d}x} + \frac{1}{8k_m} \int_{(k_F - k_m)/2}^{(k_F + k_m)/2} \frac{[k \cdot g_\alpha^{JST}(k)]^2 [k_F^2 - (2k - k_m)^2] \, \mathrm{d}k}{1 + \frac{2}{\pi}\sigma_\alpha^{JST} \int_0^\infty \frac{[x \cdot g_\alpha^{JST}(x)]^2 \bar{Q}(\bar{K}, x)}{x^2 + \gamma^2} \, \mathrm{d}x} \right\} .$$

This single-particle potential $U(k_m)$ was determined self-consistently and only few iterations were necessary to achieve convergence.

In the present calculation we include the contribution of all partial waves up to $\alpha = 2$.

In Fig. 1 we present the saturation curve for the studied potentials. They saturate at $k_F = 1.68\ \mathrm{fm}^{-1}$ with $E/A = -13.16$ MeV. The compressibility at the saturation density is 125 MeV.

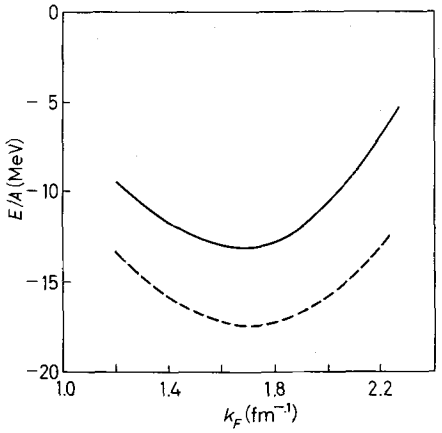

Fig. 1. – ——— Saturation curve (binding energy per particle plotted *vs.* k_F). – – – Saturation curve for the ${}^3S_1 + {}^3D_1$ channel.

A comparison of our results with the values obtained with other potentials is presented in Table I, where we find that the present potentials give at least as good results concerning nuclear matter as the ones obtained using more complicated interactions. We note that the contribution of the ${}^3S_1 + {}^3D_1$ channel by itself saturates.

TABLE I. – *The binding energy corresponding to* $k_F = 1.36\ \mathrm{fm}^{-1}$, *the saturation density and the corresponding binding energy are tabulated for different local and separable potentials.*

Potential	E/A for $k_F = 1.36\ \mathrm{fm}^{-1}$	Saturation k_F (fm^{-1})	Saturation E/A (MeV)	References
Reid H. C.	— 5.6	—	—	CLEMENT *et al.* [3]
Reid S. C.	—11.1	1.46	—12	WONG and SAWADA [4]
Hamada-Johnston	— 6.8	1.26	— 7.1	WONG and SAWADA [4]
Tabakin (rank 2)	—14.7	2	—21.15	CLEMENT *et al.* [3]
Mongan 1	—17.48	1.6	—19.3	CLEMENT *et al.* [3]
Hamman	—27.26	—	—	CLEMENT *et al.* [3]
Present potential	—11.28	1.68	—13.6	

We conclude that the potential reproduces fairly well nuclear-matter properties. These properties, coupled with the simplicity of its separable form, make it desirable for finite-nuclei calculations.

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([4]) C. W. WONG and T. SAWADA: *Ann. of Phys.*, **72**, 107 (1972).