Weak detonations from baryonic matter to strange quark matter

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Resumen / Consideramos el escenario en el cual materia de bariones se transforma en materia extraña en el núcleo de una estrella de neutrones. Suponemos que la combustión ocurre en una zona estrecha que puede ser modelada como una discontinuidad matemática. Prestamos especial atención al mecanismo de combustión conocido como detonación, y mostramos que para altas presiones pueden ocurrir detonaciones débiles. Este resultado permite calcular el estado del fluido detrás del frente de combustión en función de la densidad delante del frente para rangos de densidad más amplios que en trabajos previos.

Abstract / We consider the scenario in which baryonic matter burns to strange quark matter in the core of a neutron star. We suppose that the combustion occurs in a narrow zone which can be modeled as a mathematical discontinuity. We pay special attention to the combustion mechanism of detonation, and we show that for high pressures weak detonations can be reached. This result allows to calculate the state of the fluid after the combustion front as a function of the density before the combustion front for wider density ranges that in previous works.

Keywords / stars: neutron — dense matter — shock waves — supernovae: general

1. Introduction

It has been proposed that strange quark matter is the ground state of the matter (Witten, 1984), but this state can only be reached under extreme conditions that can be found in some supernovae or neutron stars cores. The phase transition from baryonic matter to strange quark matter in a neutron star must release about 10^{47} J of energy, so it could be the engine of some high luminosity astrophysical phenomena (Ouyed et al., 2002). But the reaction rate is unknown—proposals go from the diffusive regime to detonation—, as it depends on physical parameters of uncertain values.

There are many proposals of equations of state (EOSs) for baryon and strange quark matter. In this work we consider the polytropic EOS BJ1 by Bethe & Johnson (1974) for the baryonic matter

$$\begin{cases} P = K\, n_B^{\gamma} \\ E = m_B\, n_B + \dfrac{P}{\gamma - 1} \end{cases} \tag{1}$$

being P the pressure, E the energy density, n_B the baryon density, m_B the baryon rest energy, $\gamma = 2.54$ and $K = 364$ if $[P] = [E] = \mathrm{MeV\,fm^{-3}}$ and $[n_B] = \mathrm{fm^{-3}}$.

On the other hand, we consider the MIT Bag Model for mass-less non-interacting particles for the strange quark matter (Cleymans et al., 1986):

$$\begin{cases} P = \dfrac{1}{c^3\,\hbar^3}\left(\dfrac{19}{36}\,\pi^2\,T^4 + \dfrac{3}{2}\,T^2\,\mu^2 + \dfrac{3}{4\pi^2}\,\mu^4\right) - B \\ E = 3\,P + 4\,B \\ n_B = \dfrac{1}{c^3\,\hbar^3}\left(T^2\,\mu + \dfrac{1}{\pi^2}\,\mu^3\right) \end{cases} \tag{2}$$

being T the temperature and μ the chemical potential, both in energy units. We use $B = 60\,\mathrm{MeV\,fm^{-3}}$ as the bag constant (Farhi & Jaffe, 1984). These two EOSs, besides being simple, are supported by the M-R relation observed for neutron stars (Steiner et al., 2013). Later we will see that the results obtained by using more sophisticated EOSs should be qualitatively the same.

The energy per baryon is always lower for strange quark matter described within the MIT Bag Model (Haensel et al., 2007), but only a fraction of the baryonic matter of a neutron star can be transformed to strange quark matter (Benvenuto et al., 1989). That depends on the progenitor star structure and the burning dynamics. So, the result of the star transformation is an hybrid star with a strange quark matter core enveloped by baryonic matter.

Currently there are no conclusive results regarding the duration of the transformation (Haensel et al., 2007). If the combustion occurs in the diffusive regime, a timescale of about 10 min is expected, but under certain conditions it could be as low as 0.1 s. On the other hand, some authors argue that the instabilities in the combustion front must speed it up. If deflagration to detonation transition does not occur, the timescale of the conversion is about 10 ms. But if the detonation scenario takes place, the timescale results ~0.1 ms.

In any case, if the combustion is fast enough, it must occur in a narrow zone that can be mathematically approximated by a discontinuity. The theory of these reactions, developed by Ya. B. Zeldovich in 1940 (Landau & Lifshitz, 1959), can be extended to the relativistic case considering the Taub adiabat (Benvenuto et al., 1989).

2. The detonation mechanism

A discontinuity in a relativistic fluid must satisfy the Taub equation (Thorne, 1973)

$$E_2\,X_2 - E_1\,X_1 = P_2\,X_1 - P_1\,X_2 \tag{3}$$

where $X_i = \mu_i\,V_i$ is called *generalized volume* (Lugones et al., 1994), being $V_i = n_{Bi}^{-1}$. We use subscript 1 for the fluid immediately before the discontinuity, and subscript 2 for the fluid immediately after it. From Eq. 3 and an initial state (X_1, P_1)—which corresponds to baryon matter—and considering no phase transition—i.e. the matter after the front is also described by EOS 1—a function $P_2(X_2)$ known as shock adiabat results defined. But when the shock triggers a phase transition subscript 2 corresponds to strange quark matter. The function $P_2(X_2)$ obtained from Eq. 3, EOS 2 and the initial state (X_1, P_1) is called detonation adiabat.

In the detonation process, the unburnt fluid at state 1 is heated up by the shock wave to state 1' on the shock adiabat. Then, the fluid evolves from state 1' to state 2 on the detonation adiabat due to the combustion. Fig. 1 shows the path follows by the detonation process for an initial state of $n_B = 0.35\,\mathrm{fm}^{-3}$. The evolution from state 1' to state 2 follows the segment that connects 1' with 1, up to state 2 on the detonation adiabat is reached. There is a special final state for which the segment that joins state 1 and 2 is tangent to the detonation adiabat. This is the *Chapman-Jouguet* point, and it is the final state of self-sustained detonations, which are the only ones that occur in nature. The detonation process shown in Fig. 1 has the Chapman-Jouguet point as final state. The mechanism of deflagrations is different, but the final state must also lie on the detonation curve. For detonations $X_2 < X_1$ and $P_2 > P_1$, and for deflagrations $X_2 > X_1$ and $P_2 < P_1$. Deflagrations also have Chapman-Jouguet points which are not usually the final state, but they represent the strongest possible deflagrations with the highest velocity of propagation. On the other hand, the weakest deflagrations are these in which the reaction rate is as low as possible. We call *slow combustion limit* to the deflagrations with ΔP and velocity of propagation zero, which of course cannot be reached if the combustion occurs in a narrow zone.

But there is a peculiarity of the detonations of baryonic matter to strange quark matter: for some initial conditions, the shock and the detonation adiabats cross each other between the corresponding Chapman-Jouguet point and the initial state. In these situations the final state cannot be the Chapman-Jouguet point, but it must be the crossing point itself. Fig. 2 shows the case for $n_B = 0.75\,\mathrm{fm}^{-3}$. This occurs because the adiabatic index of strange quark matter varies significantly with the baryon density from infinity at $n_B \sim 0.287\,\mathrm{fm}^{-3}$ to 4/3 at high density, while the adiabatic index of baryonic matter does not—it is constant in this case—. In particular, this allows the final state could reach the weak detonations branch of the detonation adiabat, which is usually forbidden. The goal of this paper is to highlight this issue which is not discussed in previous works on detonations of baryonic matter to strange quark matter.

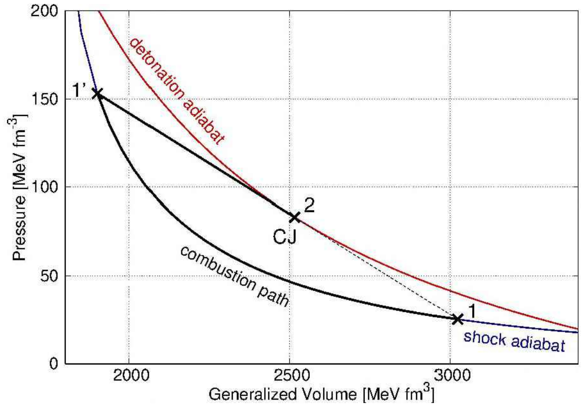

Figure 1: Shock adiabat, detonation adiabat and the path from initial state 1 to final state 2 for $n_B = 0.35\,\mathrm{fm}^{-3}$. In this case the final state is the Chapman-Jouguet point.

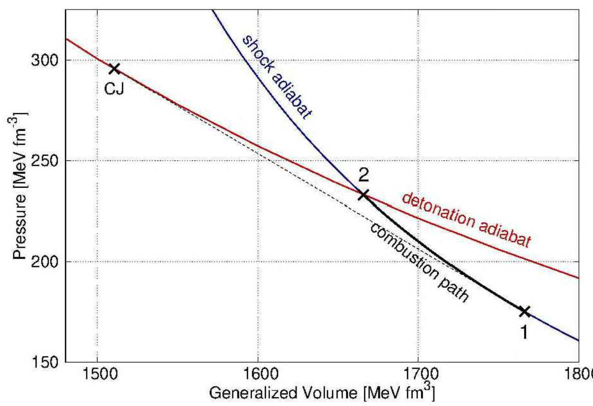

Figure 2: Shock adiabat, detonation adiabat and the path from initial state 1 to final state 2 for $n_B = 0.75\,\mathrm{fm}^{-3}$. Here the final state is the crossing point between the adiabats.

3. Results and discussion

Fig. 3 shows the numerical results for the pressure of the strange quark matter after the combustion front as function of the baryon density of the baryonic matter before it, for detonations and deflagrations. As detonations in nature have well defined final states that depend only on initial states, they are represented by a single curve. On the other hand, the final state of deflagrations depends on several physical factors that determine the rate of the reaction, but it must be limited by two curves: the slow combustion limit curve—the weakest case—, and a the curve corresponding to the strongest possible deflagrations.

We can see that the three curves are defined into an interval of baryon densities between 0.297 and $0.839\,\mathrm{fm}^{-3}$. Outside this interval the reaction is not longer exothermic, so it results inhibited (Lugones et al., 1994). We can see that the curves for detonations and the strongest deflagrations have one breaking point each one: at $n_B = 0.563\,\mathrm{fm}^{-3}$ for the former and $n_B = 0.523\,\mathrm{fm}^{-3}$ for the later. For detonations to the left of the breaking point the final states are obtained from the Chapman-Jouguet points, and for detonations to the right of the breaking point, from the crossing points between the shock and detonation adiabats. In this particular case, the branch of the curve corresponding to the crossing points has constant a pressure of $233\,\mathrm{MeV\,fm^{-3}}$. The curve for the strongest deflagrations is similar, but now the branch corresponding to the crossing points is to the left, and the branch corresponding to the Chapman-Jouguet points is to the right. The branch of the curve corresponding to the crossing points has a constant pressure of $16.7\,\mathrm{MeV\,fm^{-3}}$.

Fig. 4 shows the results for the velocity of the front for detonations, and the fastest deflagrations—the strongest ones—as function of the baryon density before it. As we stated before, the lower limit of propagation velocity for deflagrations is zero. Fig. 4 also shows the sound velocity in baryonic matter and the velocity of the strange quark matter with respect to the front, for comparison. As we can see, detonations are always supersonic and deflagrations are always subsonic.

Now the question is what happens if different—and more realistic—EOSs are considered. If the baryonic matter EOS is too stiff, the phase transition is not longer exothermic and it results inhibited (Lugones et al., 1994). But if the phase transition results exothermic only into a pressure interval, there are arguments to support that the behaviour shown here should happen. On one hand, it has been suggested that all the EOSs for strange quark matter in compact stars are well fitted by a generic linear form on only two parameters (Haensel et al., 2007). So, the dependence of the adiabatic index on density is qualitatively the same for all of them, particularly, it results infinity at $n_B \sim 0.3\,\mathrm{fm^{-3}}$. On the other hand, the adiabatic index of most accepted realistic EOSs for npeμ matter actually depends on pressure, but its value remains between 2 and 4 for baryon densities between 0.1 and $1.5\,\mathrm{fm^{-3}}$ (Haensel et al., 2007). Therefore, it is expected that for several different choices of EOSs the corresponding shock and detonation adiabats cross each other inside the baryon density interval at which the phase transition is possible. So, the behaviour shown in this paper—in which weak detonations result possible for phase transitions from baryonic matter to strange quark matter—should be typical.

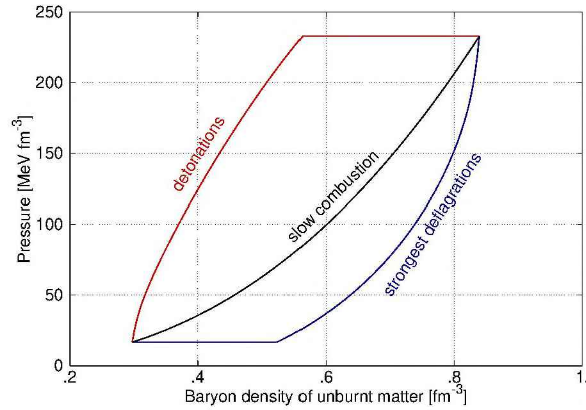

Figure 3: Pressure of the strange quark matter after the combustion front for detonations and deflagrations, as function of the baryon density of the baryonic matter before it. While the pressure for detonations is represented by a single curve, deflagrations can have pressures into an interval limited by the strongest deflagrations and the slow combustion curves.

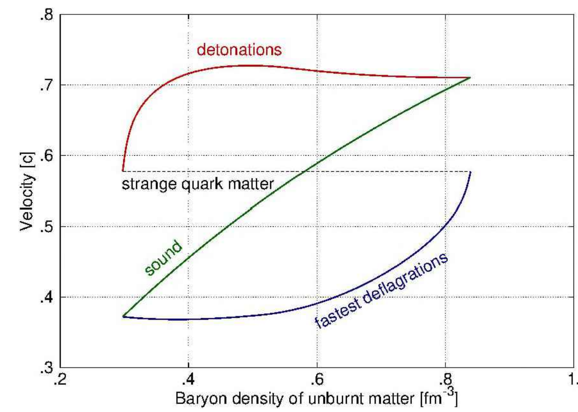

Figure 4: Velocity of the combustion front for detonations and the fastest deflagrations, and velocity of sound in baryonic matter. The velocity of the slowest deflagrations is zero. The first two curves also represent the velocity of the incoming baryonic matter with respect to the combustion front. The velocity of the outcoming strange quark matter is shown by the dashed segment.

Conclusions

We show that for typical equations of state for baryonic matter and strange quark matter, the shock and the detonation adiabats cross each other before the Chapman-Jouguet points on the detonations adiabat for some initial conditions. This fact forbids that the corresponding Chapman-Jouguet point can be reached in the combustion process, so it can no longer be the final state, which is now the crossing point itself. As a consequence, the final states for detonations at high pressure lie on the branch of weak detonations. On the other hand, the same occurs to the strongest limit for deflagrations at low pressure, which now is weaker than the Chapman-Jouguet condition. We conclude that this behaviour must be typical for phase transitions from baryonic matter to strange quark matter that are modeled as mathematical discontinuities.

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