

# Tidal evolution of a close-in planet with a more massive outer companion

A. Rodríguez<sup>1</sup>, S. Ferraz-Mello<sup>1</sup>, T. A. Michtchenko<sup>1</sup>, C. Beaugé<sup>2</sup> and O. Miloni<sup>3</sup>

<sup>1</sup>Instituto de Astronomia, Geofísica e Ciências Atmosféricas, Rua do Matão 1226, 05508-900 São Paulo, Brazil  
email: [adrian@astro.iag.usp.br](mailto:adrian@astro.iag.usp.br)

<sup>2</sup>Observatorio Astronómico, Universidad Nacional de Córdoba, Laprida 854, (X5000BGR) Córdoba, Argentina

<sup>3</sup>Facultad de Ciencias Astronómicas y Geofísicas, Universidad Nacional de La Plata, Paseo del Bosque S/N B1900 FWA, La Plata, Argentina

**Abstract.** We investigate the motion of a two-planet coplanar system under the combined effects of mutual interaction and tidal dissipation. The secular behavior of the system is analyzed using two different approaches, restricting to the case of a more massive outer planet. First, we solve the exact equations of motion through the numerical simulation of the system evolution. We also compute the stationary solutions of the mean equations of motion based on a Hamiltonian formalism. An application to the real system CoRoT-7 is investigated.

**Keywords.** Celestial mechanics

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## 1. Introduction

The tidal effect produces orbital decay, circularization and spin-orbit synchronization of the orbit of a close-in planet orbiting a slow-rotating star (Goldreich & Soter 1966; Ferraz-Mello *et al.* 2008). When the tidally affected planet has an eccentric companion the inner planet eccentricity is excited, resulting in a rapid migration rate toward the star due to tidal dissipation (Mardling & Lin 2004). Here, we study the coupled tidal-secular evolution of a system with a close-in planet and its outer more massive companion.

## 2. The model

We work with systems in which the inner planet is a super-Earth with a more massive outer companion. We assume that only the inner planet is deformed under the tides raised by the central star. The reference frame chosen is centered in the star and the motion of the planets occurs in the reference plane (i.e. coplanar motion).

The inner planet evolves under the combined effects of the interaction with the outer planet and tides raised by the star. The tidal force  $\mathbf{f}$  acting on the inner mass  $m_1$  is given by Mignard (1979):

$$\mathbf{f} = -3k_1 \Delta t_1 \frac{Gm_0^2 R_1^5}{r_1^{10}} [2\mathbf{r}_1(\mathbf{r}_1 \cdot \mathbf{v}_1) + r_1^2(\mathbf{r}_1 \times \boldsymbol{\Omega}_1 + \mathbf{v}_1)], \quad (2.1)$$

where  $\mathbf{v}_1 = \dot{\mathbf{r}}_1$  and  $\boldsymbol{\Omega}_1$  is the rotation angular velocity of the inner planet.  $\Delta t_1$  is the *time lag* and it can be interpreted as a delay in the deformation of the tidally affected body due to its internal viscosity.

### 3. Secular dynamics of two-planets system

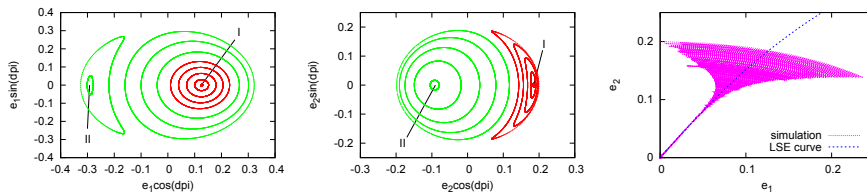
The secular behavior of the system can be also explored through the investigation of the mean dynamics (see Michtchenko & Ferraz-Mello 2001). The *mean equations of motion* of the one degree of freedom system are given by

$$\dot{I}_1 = -\frac{\partial H_{Sec}}{\partial \Delta\varpi}, \quad \Delta\dot{\varpi} = \frac{\partial H_{Sec}}{\partial I_1}, \quad (3.1)$$

where  $\Delta\varpi$  and  $I_1 = m'_i \sqrt{\mu_i a_i} (1 - \sqrt{1 - e_1^2})$  are the action-angle variables of the problem, while  $H_{Sec} = -(Gm_1 m_2 / a_2) \times R_{Sec}(L_i, K_i, \Delta\varpi)$  is the secular part of the Hamiltonian and  $R_{Sec}$  is the corresponding disturbing function. *Stationary solutions* are given by the roots of Eqns. (3.1), where the second one provides  $\Delta\varpi = (0, \pi)$  and are known as Mode I and Mode II, respectively. Periodic solutions of the secular averaged problem are motions around Mode I or Mode II (see Fig. 1). The calculation of stationary solutions can be extended to the case in which dissipation is included. Indeed, tidal dissipation produces orbital decay and thus energy loss due to internal friction. Stationary solutions of Eqns. (3.1) can be found for a range of  $a_1$  values, resulting in a curve in the space  $(e_1, e_2)$ . This curve is composed by the collection of all equilibrium points for the whole range of  $a_1$ , and we refer it as LSE curve (locus of stationary solutions in the space of eccentricities).

### 4. Application to CoRoT-7 system

CoRoT-7 planetary system is composed by two short-period super-Earth-like planets orbiting a central Sun-like star of  $m_0 = 0.93m_\odot$ . The inner and outer planets, CoRoT-7b and CoRoT-7c, are assumed to have masses of  $m_1 = 4.8m_\oplus$  and  $m_2 = 8.4m_\oplus$ , respectively. Their orbital periods are 0.854 and 3.698 days and both orbits are circular (Queloz *et al.* 2009).



**Figure 1.** *Left panels:* Secular variation of eccentricities and  $\Delta\varpi$  in the CoRoT-7 system for several initial conditions. The locations of equilibrium points are marked by I and II on each plane. *Right panel:* Comparison between LSE curve and the result of numerical simulation for CoRoT-7 system (mode I).

The result of the numerical simulation of the system including tidal dissipation (exact equations) is shown in Fig. 1, and compared with the LSE curve. Initial conditions were  $a_1 = 0.0190$  AU,  $e_1 = 0$ ,  $e_2 = 0.2$  and  $\Delta\varpi = 0$ , while  $\Delta t = 3$  min,  $R_1 = 1.68R_\oplus$ . The eccentricities oscillate rapidly with damped amplitudes around the curve of stationary solutions. Note that the LSE curve can be considered as the "mean secular path" of the system, providing thus information about the past and future dynamical behavior.

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