

# Spin dependence of heavy quark fragmentation

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## Abstract

We propose that the non-perturbative part of the fragmentation function describing the transition from a heavy quark to a heavy meson is proportional to the square of the produced meson wave function at the origin, taking into account hyperfine interactions. We analyze the effects of this proposal on the number of pseudoscalar mesons compared to the number of vector mesons produced and find a good agreement with experimental data. Finally, we discuss further experimental checks for our hypothesis.

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Heavy quark production in high energy collisions, either  $e^+e^-$ ,  $ep$ ,  $pp$  or  $p\bar{p}$ , provides a good laboratory to test QCD in both, its perturbative and non-perturbative sectors. In all the experiments both sectors contribute. In processes with a hadron in the initial state one has to consider the quark and gluon distribution functions in the initial hadron and the heavy quark fragmentation function describing the transition of the heavy quark into the measured final state hadronic system. In  $e^+e^-$  processes only this last non-perturbative piece contributes. The experimental situation on charm and bottom production in different collision events has been reviewed in [1]. It is clear that in order to get the maximum theoretical information from the increasingly precise experimental data on heavy quark production one must have a good description of all the pieces involved in the calculation, in particular of the fragmentation functions.

The heavy quark fragmentation function is obtained by the convolution of two contributions: a perturbative and a non-perturbative one. The perturbative part describes the production of a heavy quark surrounded by a cloud of soft gluons and eventual hard radiation processes. The non-perturbative part describes the hadronization of this quark with the given momentum fraction into a meson. Usually, a purely phenomenological parametrization is used for the non-perturbative part.

The most commonly used [2] are the ones by: Kartvelishvili et al. [3], Bowler [4], Peterson et al. [5], Collins and Spiller [6], Colangelo and Nason [7] and Braaten et al. [8]. Among these parametrizations only the last one [8] distinguishes among the spin of the produced meson. Here we will discuss an alternative way of making this distinction.

All these parametrizations provide different realizations of the original Bjorken [9] and Suzuki [10] proposal that the heavy quark fragmentation function, contrary to what happens with light quarks, should be very hard, i.e., the heavy quark should retain most of its momentum in the hadronization process. The exact shape of the dependence of the fragmentation function on the heavy quark momentum is controlled, in the parametrizations cited above, by some free parameters (the number of free parameters depends on the parametrization) that have to be fitted to the experimental data. In general these parameters have no absolute physical meaning (see for example [2] for a discussion).

In addition to the dependence of the fragmentation functions with the heavy quark momentum, the different experiments have also measured the relative number of charmed and bottom mesons produced in the pseudoscalar state ( $D$  and  $B$ ) and in the vector state ( $D^*$ ,  $B^*$ ). More specifically, they have measured the quantity  $P_V$  defined by

$$P_V = \frac{V}{P + V}, \quad (1)$$

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where  $V$  and  $P$  stand for the number of vector and pseudoscalar states produced, respectively. The present experimental values for charmed mesons are:

$$P_V^D = 0.566 \pm 0.025_{-0.022-0.023}^{+0.007+0.022}, \quad \text{ZEUS } (\gamma p) \quad [11],$$

$$P_V^D = 0.590 \pm 0.037_{-0.018}^{+0.022}, \quad \text{ZEUS (DIS)} \quad [12],$$

$$P_V^D = 0.693 \pm 0.045 \pm 0.004 \pm 0.009, \quad \text{H1} \quad [13],$$

$$P_V^D = 0.614 \pm 0.023, \quad e^+e^- \text{ average} \quad [14].$$

These values provide a world average

$$\bar{P}_V^D = 0.611 \pm 0.016. \quad (2)$$

For  $B$  mesons there is only one measurement available [1]

$$P_V^B = 0.75 \pm 0.04_{-0.025}^{+0.023}. \quad (3)$$

The values for the charmed mesons are clearly smaller than the widely used naive spin counting prediction  $P_V = 0.75$ , but this is not the case for the bottom mesons. So, whatever mechanism in the fragmentation process is claimed to be responsible for the decrease of  $P_V$  with respect to the naive prediction should, in a natural way, produce a much smaller effect for mesons containing a  $b$  quark than for mesons containing a  $c$  quark.

In this Letter we propose that the fragmentation function of a heavy quark,  $Q$ , into a heavy meson  $M$  should be proportional to the square of the meson  $M$  hyperfine-corrected wave function at the origin. It is important to notice that in the analysis of semileptonic meson decays a sensible improvement of the theoretical results when compared with experimental data, was obtained by breaking heavy quark symmetry and taking hyperfine interactions into account [15]. We, thus, propose to explicitly include in the naive  $P_V$  expression (1) the hyperfine corrections via different wave functions at the origin for scalar and vector states. Consequently, we consider instead

$$P_V = \left( \frac{P}{V} + 1 \right)^{-1} = \left( \frac{1}{3} \frac{|\psi_P(0)|^2}{|\psi_V(0)|^2} + 1 \right)^{-1}. \quad (4)$$

Notice that if the wave functions at the origin are known, our expression (4) does not include new unknown parameters because we only modify the normalization of the fragmentation functions leaving their dependence on the heavy quark momentum unchanged.

Even if at lowest order in Heavy Quark Effective Theory the wave functions at the origin for the pseudoscalar and vector states are the same, and this would reproduce the naive spin counting result, they differ at  $O(1/m_Q)$  and, obviously, the effects on the  $P_V$  predictions will be larger for mesons containing a  $c$  quark than for mesons containing a  $b$  quark.

In the update [15] of the Isgur–Scora–Grinstein–Wise model [16] one finds approximate variational wave functions that consider separately each spin state. The distinction between spin states is mandatory in order to get agreement with the experimental data for the decays. The wave functions for the lowest lying pseudoscalar and vector states are taken to be of the form

$$\psi_{1S} = \frac{\beta_S^{3/2}}{\pi^{3/4}} \exp\left(-\frac{1}{2}\beta_S^2 r^2\right), \quad (5)$$

Table 1

Values of the parameter  $\beta_S$  entering in the wave functions of the mesons, see Eq. (5) and predictions for heavy meson production rates

Meson	Mass (MeV)	$\beta_S$ (GeV)	$P_V$
$D$	1864.1	0.45	0.64
$D^*$	2006.7	0.38	
$D_s$	1969.0	0.56	0.59
$D_s^*$	2112.0	0.44	
$B$	5279.3	0.43	0.71
$B^*$	5325.0	0.40	
$B_s$	5369.6	0.54	0.69
$B_s^*$	5412.8	0.49	

where the parameter  $\beta_S$  is fixed in [15] minimizing the energies, including hyperfine interactions, to properly describe the observed meson spectra. It is important to notice that the values obtained in this way for the different meson states provide a good description of all semileptonic  $c$  and  $b$  decays. The values obtained in Ref. [15] are shown in Table 1. Since the wave function is an intrinsic property of the meson, one can use these results for the production process. This is done, for instance, in  $J/\Psi$  production in  $e^+e^-$  where the cross section is written in terms of the decay width  $\Gamma(J/\Psi \rightarrow e^+e^-)$  which is proportional to the  $J/\Psi$  wave function at the origin. Also, in the calculation of the  $J/\Psi$  fragmentation function, the wave function at the origin is fixed from the data on the  $J/\Psi$  decay width [17].

The results we obtain plugging the hyperfine-corrected wave functions at the origin from Eq. (5) into our Eq. (4) are shown in the last column of Table 1. We obtain a sensible reduction in the value of  $P_V^D$  with respect to the naive spin counting prediction and our result is in good agreement with the experimental data. For  $P_V^B$  the obtained reduction is much smaller and the result is within one standard deviation of the experimental data, Eq. (3). We should stress here once more that these values of  $P_V$  only depend on one tunable parameter for each meson,  $\beta_S$ . For these parameters we have taken the values obtained in Ref. [15] in a completely independent analysis, as discussed above.

We have also calculated our predictions for the relative number of  $D_s$  and  $D_s^*$  as well as  $B_s$  and  $B_s^*$  and shown the results in the last column of in Table 1. The reduction with respect to the naive prediction is larger in the mesons containing a heavy and a strange quark than in the mesons containing a heavy and a  $u$  or  $d$  quark. This is a prediction of our assumption that can be experimentally checked. In particular, the low value of  $P_V^{D_s}$  looks promising for such a test.

The expression in Eq. (4) is only valid if both the pseudoscalar and vector mesons entering have the same quark content. If only one of them contains a strange quark (in addition to the heavy quark) one has to take into account also the strange suppression factor that is usually defined as the ratio of the number of charmed strange mesons with respect to the number of charmed non-strange mesons produced. However, under our hypothesis one should be careful because these ratios depend now on the hyperfine-corrected wave functions at the

origin. One would expect, however, that the strange suppression factor would only contain information about the relative probability of producing an  $s\bar{s}$  from the vacuum compared to the probability of producing a pair of light quarks. This means, one would expect the strange suppression factor to be independent of the spin of the produced mesons. So, in order to define such a spin independent strange suppression factor one has to use:

$$\gamma_{SI} = \frac{|\psi_D(0)|^2}{|\psi_{D_s}(0)|^2} \gamma_P = \frac{|\psi_{D^*}(0)|^2}{|\psi_{D_s^*}(0)|^2} \gamma_V, \quad (6)$$

where  $\gamma_P = D_s/D$  and  $\gamma_V = D_s^*/D^*$  are the suppression factors measured in the pseudoscalar and vector channels, respectively. Using the  $\beta_S$  parameters listed in Table 1, it is clear that  $\gamma_{SI} = 0.52\gamma_P = 0.64\gamma_V$ .

### Final remarks

The naive spin counting prediction for the ratio  $P_V$  does not fit the experimental data, in particular in the case of charmed  $D$  mesons. We propose that the fragmentation functions should be proportional to the square of the produced meson hyperfine-corrected wave function at the origin. This amounts to change the normalization factors of the available non-perturbative fragmentation functions keeping their dependence on the heavy-meson energy fraction unchanged. We have analyzed the effects of this proposal on the values of  $P_V$  for charmed and bottom mesons and found very good agreement with the experimental data using values for the wave functions at the origin fixed from meson decays. As a way to check our proposal we estimate the values of  $P_V$  for charmed-strange and bottom-strange mesons for which no experimental data are yet available. Further checks can be performed measuring our proposed spin dependence of the strange suppression factor.

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