Neutrino and antineutrino cross sections in ^{12}C

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Abstract. We extend our formalism of weak interaction processes, obtaining new expressions for the transition rates, which greatly facilitate numerical calculations, of neutrino (ν) and antineutrino ($\tilde{\nu}$)-nucleus reactions and the muon capture. We have done a thorough study of exclusive (ground state) properties of ^{12}B and ^{12}N within the Projected Quasiparticle Random Phase Approximation (PQRPA). Good agreement with experimental data is achieved in this way. The inclusive $\nu/\tilde{\nu}$-nucleus reactions ^{12}C$(\nu, e^-)^{12}$N and ^{12}C$(\tilde{\nu}, e^+)^{12}$B are calculated within both the PQRPA, and the Relativistic QRPA (RQRPA). It is found that the magnitudes of the resulting cross-sections: i) are close to the sum-rule limit at low energy, but significantly smaller than this limit at high energies both for ν and $\tilde{\nu}$, ii) steadily increase when the size of the configuration space is augmented, and particulary for $\nu/\tilde{\nu}$ energies > 200 MeV, and iii) converge for sufficiently large configuration spaces and final state angular momenta.

1. Introduction

The neutrino-nucleus scattering on ^{12}C is important because this nucleus is the basic ingredient of many liquid scintillator detectors. As such it has been employed to search for neutrino oscillations, and for measuring neutrino-nucleus cross sections with experimental facilities LSND, KARMEN, and LAMPF. The ^{12}C target will be used as well in several planned experiments, such as the spallation neutron source (SNS) at Oak Ridge National Laboratory, and the Large Volume Detector (LVD) at Gran Sasso National Laboratories. On the other hand, this nucleus is important for astrophysics studies, as it forms one of the onion-like shells of large stars before they collapse. Concomitantly, the LVD group have stressed recently the importance of measuring supernova neutrino oscillations. A comprehensive theoretical review on neutrino-nucleus interaction was done by Kolbe *et al.* [1].

2. Formalism for the Weak Interacting Processes

The most widely used formalism for neutrino-nucleus scattering was developed by the Walecka's group [2]. They classify the nuclear transition moments as Coulomb, longitudinal, transverse electric, and transverse magnetic. This terminology, however, is not used in nuclear β-decay and μ-capture, where one only speaks on vector and axial matrix elements with different degrees of forbiddenness: allowed (GT and Fermi), first forbidden, second forbidden, *etc.* Motivated by this fact, we have extended the formalism for weak interaction processes developed in [3, 4], obtaining new expressions for the transition rates, which greatly facilitate numerical calculations, both for neutrino-nucleus reactions and muon capture [5].

The weak Hamiltonian is expressed in the form [2]

$$H_W(\mathbf{r}) \;=\; \frac{G}{\sqrt{2}} J_\alpha l_\alpha e^{-i\mathbf{r}\cdot\mathbf{k}}, \tag{1}$$

where $G = (3.04545 \pm 0.00006)\times 10^{-12}$ is the Fermi coupling constant (in natural units). The leptonic current $l_\alpha \equiv \{\mathbf{l}, il_\emptyset\}$ is given by [4, Eq. (2.3)], and the nonrelativistic form of the hadronic current operator $J_\alpha \equiv \{\mathbf{J}, iJ_\emptyset\}$ is used [4]. The quantity $k = P_i - P_f \equiv \{\mathbf{k}, ik_\emptyset\}$ is the momentum transfer, M is the nucleon mass, and P_i and P_f are, respectively, the momenta of the initial and final nucleon (nucleus). The effective vector (V), axial-vector (A), weak-magnetism (W), and pseudoscalar (P) dimensionless coupling constants are defined in [4]. In performing the multipole expansion of the nuclear operators $O_\alpha \equiv (\mathbf{O}, iO_\emptyset) = J_\alpha e^{-i\mathbf{k}\cdot\mathbf{r}}$ it is convenient: 1) to take $\mathbf{k}$ along the z axis, *i.e.*, $e^{-i\mathbf{k}\cdot\mathbf{r}} \to e^{-i\kappa z}$, and 2) to make use of Racah's algebra. One obtains

$$\begin{aligned} O_{\emptyset\mathsf{J}} &= g_V \mathcal{M}^V_{\mathsf{J}} + i g_A \mathcal{M}^A_{\mathsf{J}} + i(\overline{g}_{\mathrm{A}} + \overline{g}_{\mathrm{P1}})\mathcal{M}^A_{0\mathsf{J}} && (2)\\ O_{m\mathsf{J}} &= i(\delta_{m0}\overline{g}_{\mathrm{P2}} - g_{\mathrm{A}} + m\overline{g}_{\mathrm{W}})\mathcal{M}^A_{m\mathsf{J}} + g_V \mathcal{M}^V_{m\mathsf{J}} - \delta_{m0}\overline{g}_{\mathrm{V}}\mathcal{M}^V_{\mathsf{J}}, && (3)\end{aligned}$$

with the elementary operators

$$\begin{aligned} &\mathcal{M}^V_{\mathsf{J}} = j_{\mathsf{J}}(\rho) Y_{\mathsf{J}}(\hat{\mathbf{r}}), && \mathcal{M}^A_{m\mathsf{J}} = \sum_{\mathsf{L}\geq 0} i^{\mathsf{J}-\mathsf{L}-1} F_{\mathsf{LJ}m} j_{\mathsf{L}}(\rho)\left[Y_{\mathsf{L}}(\hat{\mathbf{r}}) \otimes \boldsymbol{\sigma}\right]_{\mathsf{J}},\\ &\mathcal{M}^A_{\mathsf{J}} = \mathrm{M}^{-1} j_{\mathsf{J}}(\rho) Y_{\mathsf{J}}(\hat{\mathbf{r}})(\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}), && \mathcal{M}^V_{m\mathsf{J}} = \mathrm{M}^{-1}\sum_{\mathsf{L}\geq 0} i^{\mathsf{J}-\mathsf{L}-1} F_{\mathsf{LJ}m} j_{\mathsf{L}}(\rho)[Y_{\mathsf{L}}(\hat{\mathbf{r}}) \otimes \boldsymbol{\nabla}]_{\mathsf{J}}, \end{aligned} \tag{4}$$

where $F_{\mathsf{LJ}m} \equiv (-)^{1+m}(1, -m\mathsf{J}m|\mathsf{L}0)$ are Clebsch-Gordan coefficients [4]. The CVC relates the vector-current pieces of the operator O_α as $\mathbf{k}\cdot\mathbf{O}^V \equiv \kappa O^V_0 = \tilde{k}_\emptyset O^V_\emptyset$ with $\tilde{k}_\emptyset \equiv k_\emptyset - S(\Delta E_{\mathrm{Coul}} - \Delta\mathrm{M})$, where $\Delta E_{\mathrm{Coul}} \cong \frac{6e^2 Z}{5R} \cong 1.45 Z A^{-1/3}$ MeV is the Coulomb energy difference between the initial and final nuclei, $\Delta\mathrm{M} = \mathrm{M}_n - \mathrm{M}_p = 1.29$ MeV is the neutron-proton mass difference, and $S = \pm 1$ for neutrino and antineutrino scattering, respectively.

The transition amplitude $\mathcal{T}_{\mathsf{J}^\pi_n}(\kappa) \equiv \sum_{s_\ell, s_\nu} |\langle \mathsf{J}^\pi_n | H_W(\kappa) | 0^+\rangle|^2$ for the neutrino-nucleus reaction at a fixed value of κ, from the ground state $|0^+\rangle$ in the (Z, N) target nucleus to the n-th final state $|\mathsf{J}^\pi_n\rangle$ in the nucleus $(Z \pm 1, N \mp 1)$, reads

$$\mathcal{T}_{\mathsf{J}^\pi_n}(\kappa) \;=\; 4\pi G^2 [\sum_{\alpha=\emptyset,0,\pm 1} |\langle \mathsf{J}^\pi_n || O_{\alpha\mathsf{J}}(\kappa) || 0^+\rangle|^2 \mathcal{L}_\alpha - 2\Re\left(\langle \mathsf{J}^\pi_n || O_{\emptyset\mathsf{J}}(\kappa) || 0^+\rangle \langle \mathsf{J}^\pi_n || O_{0\mathsf{J}}(\kappa) || 0^+\rangle^*\right) \mathcal{L}_{\emptyset 0}], \tag{5}$$

where the momentum transfer is $k = p_\ell - q_\nu$, with $p_\ell \equiv \{\mathbf{p}_\ell, iE_\ell\}$ and $q_\nu \equiv \{\mathbf{q}_\nu, iE_\nu\}$. The lepton traces $\mathcal{L}_\emptyset, \mathcal{L}_0, \mathcal{L}_{\pm 1}$ and $\mathcal{L}_{\emptyset 0}$ are defined in [4].

3. Neutrino (antineutrino)-nucleus cross sections

The exclusive cross-section (ECS) for the state $|\mathsf{J}^\pi_n\rangle$, as a function of the incident neutrino energy E_ν, is

$$\sigma_\ell(\mathsf{J}^\pi_n, E_\nu) \;=\; \frac{|\mathbf{p}_\ell| E_\ell}{2\pi} F(Z+S, E_\ell) \int_{-1}^{1} d(\cos\theta)\, \mathcal{T}_{\mathsf{J}^\pi_n}(\kappa), \tag{6}$$

where $E_\ell = E_\nu - \omega_{\mathsf{J}^\pi_n}$ and $|\mathbf{p}_\ell|$ are the energy and modulus of linear momentum of the lepton $\ell = e, \mu$, and $\omega_{\mathsf{J}^\pi_n} = -k_\emptyset = E_\nu - E_\ell$ is the excitation energy of the state $|\mathsf{J}^\pi_n\rangle$ relative to the initial state $|0^+\rangle$. Moreover, $F(Z+S, E_\ell)$ is the Fermi function for neutrino ($S=1$), and antineutrino ($S=-1$) processes, respectively. Here, we will also deal with the inclusive cross-sections (ICS),

$$\sigma_\ell(E_\nu) = \sum_{\mathsf{J}_n^\pi} \sigma_\ell(\mathsf{J}_n^\pi, E_\nu), \tag{7}$$

as well as with folded cross-sections, both exclusive, and inclusive

$$\overline{\sigma}_\ell(\mathsf{J}_n^\pi) = \int dE_\nu \sigma_\ell(\mathsf{J}_n^\pi, E_\nu) n_\ell(E_\nu) \quad , \quad \overline{\sigma}_\ell = \int dE_\nu \sigma_\ell(E_\nu) n_\ell(E_\nu), \tag{8}$$

where $n_\ell(E_\nu)$ is the neutrino (antineutrino) normalized flux. In the evaluation of both neutrino, and antineutrino ICS the summation in (7) goes over all n states with spin and parity $\mathsf{J}^\pi \leq 7^\pm$ in the Projected Quasiparticle Random Phase Approximation (PQRPA), and over all $\mathsf{J}^\pi \leq 14^\pm$ in the Relativistic QRPA (RQRPA).

4. Numerical results

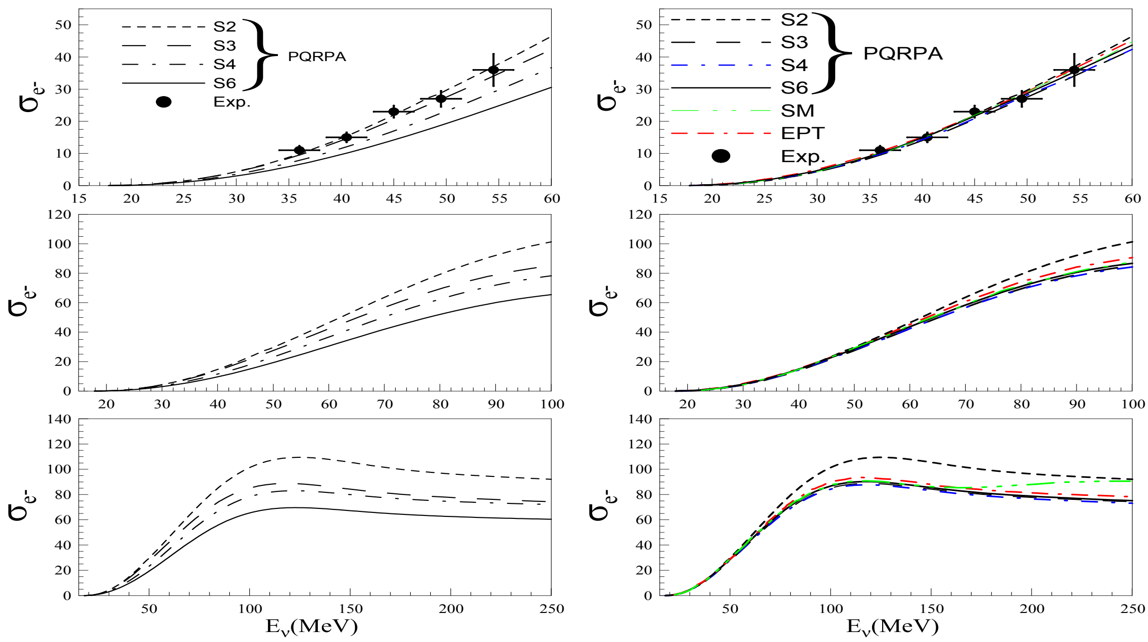

Figure 1. (Color online) Exclusive cross section $\sigma_{e^-}(1_1^+, E_\nu)$ for the reaction $^{12}\mathrm{C}(\nu_e, e^-)^{12}\mathrm{N}$ (in units of 10^{-42} cm^2), as a function of the incident neutrino energy E_ν. On the left side $t=0$ for all S_N, whereas on the right side $t=0$ for S_2, and S_3, $t=0.2$ for S_4, and $t=0.3$ for S_6. The experimental data in the DAR region are from Ref. [7].

The calculations were performed with the PQRPA using the model spaces S_2, S_3, S_4, and S_6 including 2, 3, 4, and 6 $\hbar\omega$ harmonic oscillators shells, respectively, and by employing a δ-interaction. For the first three spaces the single-particle (s.p.) energies, and pairing strengths were varied in a χ^2 search to account for the experimental spectra of odd-mass nuclei ^{11}C, ^{11}B, ^{13}C, and ^{13}N [4]. As this method can not be used for the S_6 space, which comprises 21 s.p. levels, the energies in this case were derived as in Ref. [11], while the pairing strengths were adjusted to reproduce the experimental gaps in ^{12}C [5]. We also employ the RQRPA theoretical framework [6] with S_{20}, and S_{20} spaces. In this case, the ground state is calculated in the Relativistic

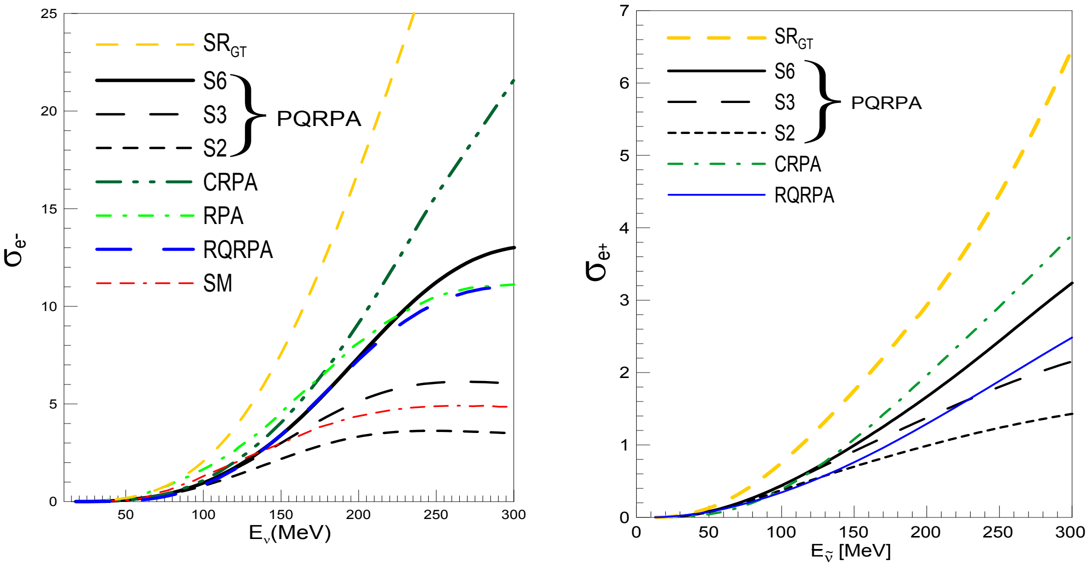

Figure 2. (Color online) Inclusive cross sections $\sigma_{e^-}(E_\nu)$, and $\sigma_{e^+}(E_{\tilde{\nu}})$ (in units of 10^{-39} cm^2) for the reactions ^{12}C$(\nu_e, e^-)^{12}$N, and ^{12}C$(\tilde{\nu}_e, e^+)^{12}$B, respectively, plotted as a function of incident neutrino and antineutrino energies. The PQRPA results within the s.p. spaces S_2, S_3, and S_6, have the same parametrization as those on the right panel of Fig. 1. The sum rule limit SR_{GT} and several previous RPA-like calculations, namely: RPA [9], CRPA [10], and RQRPA within S_{20} for two-quasipartile cutoff E_{2qp}=100 MeV [11], as well as the shell model result [9], are also exhibited.

Hartree-Bogoliubov model using effective Lagrangians with density dependent meson-nucleon couplings and DD-ME2 parameterization, and pairing correlations are described by the finite range Gogny force.

Fig. 1 shows ECS ^{12}C$(\nu_e, e^-)^{12}$N(1^+_1) plotted as a function of the incident neutrino energy E_ν for different energy zones: i) $E_\nu \leq 60$ MeV; decay at rest of π^+ (DAR) region, where the ECS was measured [7], ii) $E_\nu \leq 100$ MeV, region where the supernovae neutrino signal is expected, and iii) $E_\nu \leq 250$ MeV; decay at flight of π^+ (DIF) region, where the neutrinos oscillation search was done at LSND. On the left side the δ-interaction particle-particle strength is $t = 0$ for all spaces S_N [1], whereas on the right side t is gauged to reproduce the ground state energy of ^{12}N and the $B(GT)$ values of ^{12}N and ^{12}B, getting $t = 0$ for S_2 and S_3, $t = 0.2$ for S_4, and $t = 0.3$ for S_6.

On the left and right panels of Fig. 4 are displayed, respectively, the ^{12}C$(\nu_e, e^-)^{12}$N and ^{12}C$(\tilde{\nu}_e, e^+)^{12}$B ICS $\sigma_{e^\mp}(E_\nu)$. The PQRPA results within the s.p. spaces S_2, S_3 and S_6, have the same values of t as in the right panel of Fig. 1. They are compared with: i) the sum rule limit SR_{GT} obtained with an average excitation energy $\overline{\omega_{J^\pi_n}}$ of 17.34 MeV, ii) other RPA-like calculations, and iii) the shell model (SM).

In Fig. 3 are plotted the $\sigma_{e^\mp}(E_\nu)$, evaluated in RQRPA with different configuration spaces according the values of quasiparticle energy cut-off E_{qp}. One sees that the ICS increase as the configuration space increase, saturating at higher and higher energies.

Finally, Fig. 4 presents the RQRPA results for $\sigma_{e^\mp}(E_\nu)$ with different maximal spins J. We have found that while for antineutrinos contribute spins up to $J = 11$ only, it is needed to go

[1] This value for S_3 was adopted in Ref. [4].

up to $J = 14$ in the case of neutrinos. It is also clear that to achieve the saturation one should go to energies larger than 600 MeV.

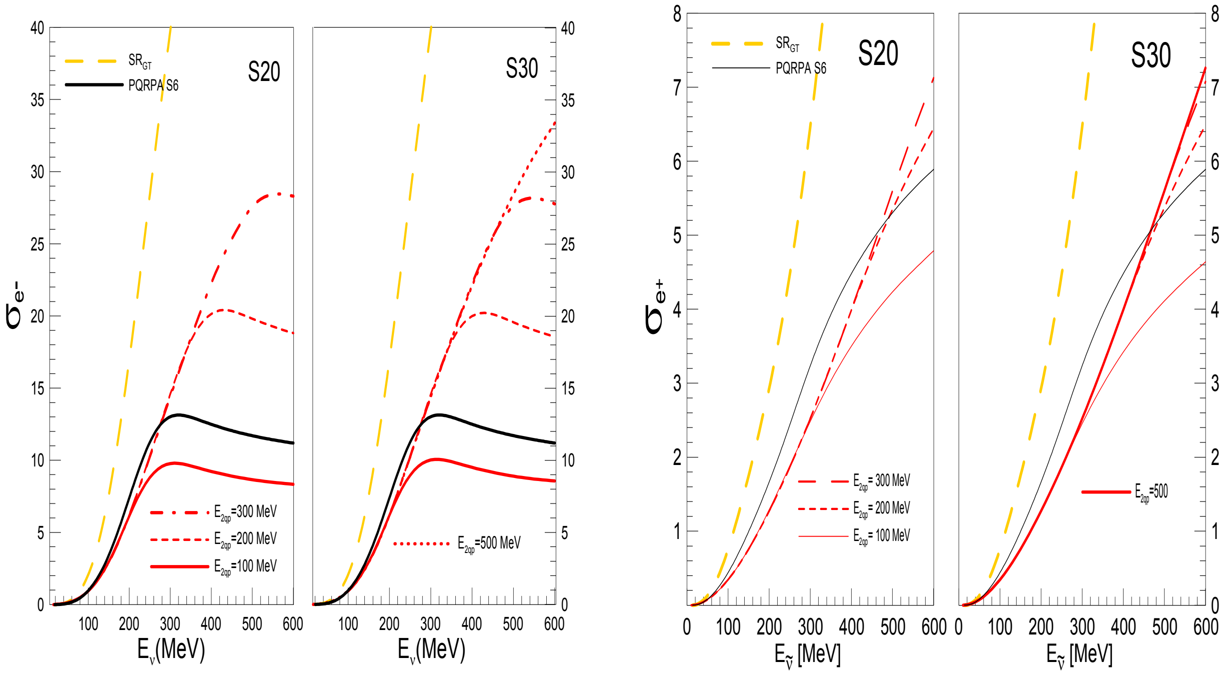

Figure 3. (Color online) Inclusive cross sections ^{12}C$(\nu_e, e^-)^{12}$N (left panel) and ^{12}C$(\tilde{\nu}_e, e^+)^{12}$B (right panel) (in units of 10^{-39} cm^2) evaluated within the RQRPA for different configuration spaces.

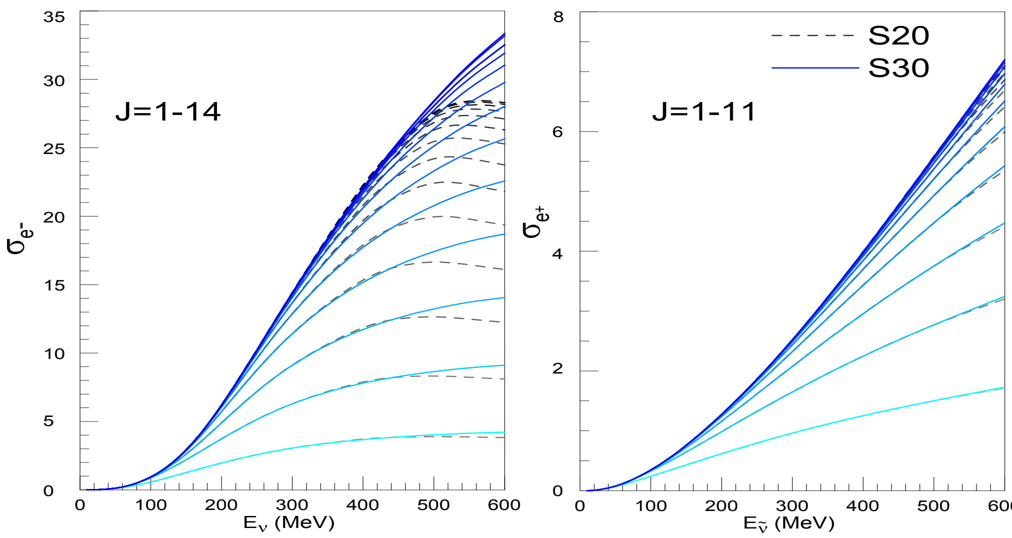

Figure 4. (Color online) Inclusive cross sections ^{12}C$(\nu_e, e^-)^{12}$N (left panel) and ^{12}C$(\tilde{\nu}_e, e^+)^{12}$B (right panel) (in units of 10^{-39} cm^2) evaluated within the RQRPA for $E_{2qp} = 500$ MeV and spaces S_{20}, and S_{30}, for different maximal spins, which go up to $J = 14$ for neutrinos, and up to $J = 11$ for antineutrinos.

5. Summary

The shell model and the PQRPA are proper theoretical frameworks to describe the ground state properties of ^{12}B and ^{12}N and the elastic cross section $\sigma_{e^-}(1_1^+, E_\nu)$ for the reaction $^{12}\mathrm{C}(\nu_e, e^-)^{12}\mathrm{N}$. The calculated inclusive cross section, at difference with the exclusive ones, steadily increase in magnitude when the size of the configuration space is augmented, and particulary for neutrino energies larger than 200 MeV. At relatively low energy they approach to the first-forbidden sum-rule limit, but at high energies they are significantly smaller both for neutrino and antineutrino. Their multipolar decomposition has been related with the proposal done in Ref. [8] to perform nuclear structure studies of forbidden processes by using low energy neutrino and antineutrino beams. We conclude that to scrutinize forbidden reactions in the $^{12}\mathrm{C}(\tilde{\nu}, e^+)^{12}\mathrm{B}$ process, one would need $\tilde{\nu}$-fluxes with $E_{\tilde{\nu}}$ up to $\gtrsim$150 MeV, while in the proposed experiments are reached energies up to 80 MeV only.

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