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$\mathcal{N} = 2$ SUSY Abelian Higgs model with hidden sector and BPS equations

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ABSTRACT: In this paper we study a system inspired on certain SUSY breaking models and on more recent Dark Matter scenarios. In our set-up, two Abelian gauge fields interact via an operator that mixes their kinetic terms. We find the extended Supersymmetric version of this system, that also generates a Higgs portal type of interaction. We obtain and study both analytically and numerically, the equations defining topologically stable string-like objects. We check our results using two different approaches.

KEYWORDS: Extended Supersymmetry, Solitons Monopoles and Instantons

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¹Also at CP3 Origins, SDU. Odense, Denmark.

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1 Introduction

Different problems in theoretical Physics suggest the existence of *hidden sectors*. These consist of $SU(3) \times SU(2) \times U(1)$ singlet fields. Indeed, since many extensions of the Standard Model propose the existence of additional product gauge groups (from Technicolor to Heterotic string inspired models), we have reasons to imagine that there are particles transforming under the new gauge fields, but not under the familiar local symmetries.

One possible way of coupling the ‘hidden’ and the ‘visible’ sectors is mediated by the Higgs field; these are called Higgs-portals. These models propose an interaction of the form $L_{\text{int}} \sim \alpha \Phi \Phi^* X X^*$, where α is the coupling, Φ is the recently measured Higgs particle and X a complex field in the hidden sector [1–3].

On the other hand, different models of Supersymmetry (SUSY) breaking also propose the existence of a hidden sector generating the dynamics that breaks SUSY, hence avoiding the constraints imposed by sum-rules on the masses of superpartners. In this case, the interest focuses on the mechanism of communication between the hidden and visible sectors. In the work [3], a mechanism was proposed that used a renormalisable interaction between two $U(1)$ gauge fields, one visible A_μ with curvature $F_{\mu\nu}$ and one hidden C_μ ,

with field strength $G_{\mu\nu}$. The interaction Lagrangian is $L_{\text{int}} \sim \xi F_{\mu\nu} G^{\mu\nu}$ and was originally proposed in [4]–[6]. This interaction can be generated at arbitrarily high energies and is not suppressed by powers of the scale, due to the marginal character of the operator. It was argued by the authors of [3] that this type of interactions are generically induced by one-loop effects and are naturally expected to appear in String theory models.

As explained in [6], gauge-gauge interactions of the form discussed above, can be diagonalised and the system becomes that of two decoupled gauge fields at the expense of charging the matter fields (hidden and visible) under both U(1)’s. This implies that the matter fields will develop a small charge — if the parameter ξ is small¹ — under the hidden and visible gauge symmetries. Astrophysical observations place strong constraints on milli-charged matter [8]. Hence, in what follows, we will consider that the hidden U(1) group is spontaneously broken. Indeed, if this hidden U(1) were not broken, ‘hidden photons’ would interfere with and spoil the process of nucleosynthesis. Also, experiments such as laser polarisation or light shining through a wall [9] tightly constrain the value of the parameter ξ .

The reader may wonder about more generic versions of the interaction, involving for example non-Abelian gauge groups. It can be seen that below the scale of breaking, the dynamics is well captured by the Abelian interaction we are discussing, see [10]. Hence, an accurate description of the low energy dynamics of our system consists of two coupled Abelian Higgs Models. This is what we will do in this paper to study the dynamics of topological objects.

Supersymmetric versions of the models with gauge-gauge interactions, like the ones we are discussing above present various appealing features. Among them, that the hidden sector will provide (if R-parity preserving) a candidate for dark matter. Hence, the dynamics of dark matter and its interaction with the Standard Model particles is another strong motivation to consider these models.

In this context, the problem of Physics that guided the present investigation is the dynamics of ‘hidden’ (or ‘dark’) strings that appear in these systems. These dark-strings [10]–[13], have a tension of the order of the symmetry breaking scale $\mu \sim (\text{TeV})^2$. They are not detectable by their gravitational effects on the CMB, but their coupling to the Standard Model fields (the milli-charged particles already mentioned) can make the dark strings observable via Bohm-Aharonov effect or by scattering of the Standard Model particles with the string core [10]. The cosmological and astrophysical signatures and consequences of these effects were studied, for example in [14, 15]. See also [16, 17] for various phenomenological aspects of these objects.

In this work, we will give a step towards the understanding of these dark strings by investigating them as BPS objects.

To summarise our motivations and framework, we have discussed the phenomenological interest (either for Beyond the Standard Model scenarios or Dark Matter models) of two type of interactions between hidden/dark and visible sectors; gauge kinetic mixing term

¹If the number ξ is not smaller than 10^{-3} this interaction seems to be ruled out experimentally [7].

and Higgs portal interactions,

$$L_{\text{GKM}} = \frac{\xi}{2} F_{\mu\nu} G^{\mu\nu}, \quad L_{\text{HP}} = -\alpha \Phi^\dagger \Phi X^* X. \tag{1.1}$$

Now, with Supersymmetry, a gauge mixing term *automatically implies* the occurrence of a Higgs portal. Indeed, the mixing of the two auxiliary fields D, D' belonging to the gauge multiplets forces a mixing of the scalars in the chiral superfields. We also need to add Fayet-Iliopoulos terms in such theories, which orchestrate, via the Higgs mechanism, the needed spontaneous breaking of the gauge groups. A discussion of new Physics arising from this mixing can be found in [18] and references therein.

As anticipated above, our system of interacting visible and hidden fields will be accurately described at observable scales by two Abelian Higgs Models coupled by a gauge kinetic mixing interaction. When these visible and/or hidden $U(1)$ gauge symmetries are spontaneously broken, the classical equations of motion have vortex solutions, which could be interpreted as visible and dark strings [10]–[13]. Properties of such vortex configurations arising in Abelian Higgs models with the gauge kinetic mixing of eq. (1.1) have been discussed in [11]–[19]. However, except for very particular cases where the Ansätze for the gauge potentials are equal [11], no first order self-dual equations have been found by establishing an energy bound “à la Bogomol’nyi [20] — see the discussion in [19].

Here we propose a different approach to this problem. As observed in [20]–[21], the classical equations of motion of the Abelian Higgs model originally discussed by Nielsen-Olesen [22] can be reduced to first-order self-duality equations when the gauge and quartic scalar self-interaction coupling constants obey a relation naturally imposed by supersymmetry. The theory then coincides exactly with a particular bosonic sector of a highly supersymmetric parent theory. The logic of this connection was understood after the work of Olive and Witten on Bogomol’nyi equations for kinks and dyons [23] and discussed in detail in the case of vortices in [24]–[26].

In more precise terms, the gauge theory with spontaneous symmetry breaking and a topological charge associated with the vortex magnetic flux, can be thought of as being part of an $\mathcal{N} = 2$ supersymmetric extension to the original model. Its energy is bounded by the $\mathcal{N} = 2$ central charge that is proportional to the topological charge induced by the vortices [24]. Furthermore, this bound is saturated when the fields obey a set of first order equations, the Bogomol’nyi-Prasad-Sommerfeld (BPS) equations. Solutions of these equations naturally have finite energy and therefore are well-suited to study objects like vortices. These arguments, formulated for just one Abelian Higgs model, extend naturally to a theory with a mixture of two such models as described earlier, with gauge kinetic mixing and Higgs portal type interactions.

It is the purpose of this work to derive such extension. The paper is organized as follows: in section 2 we present the $\mathcal{N} = 2$ supersymmetric extension of a (2+1) dimensional theory in which two Abelian Higgs models are coupled through a gauge kinetic mixing, showing how a Higgs portal interaction necessarily arises. Starting from the supersymmetry transformations, in section 3 we obtain the Bogomol’nyi equations and the energy of the system, bounded by the topological charge. In section 4 we present the $\mathcal{N} = 2$ supercharge

algebra, which explains the connection between the central charge and the visible and hidden magnetic fluxes. Both in sections 3 and 4. A careful numerical analysis of the vortex solutions is presented in section 5. An alternative derivation of the main results based on diagonalization of the supersymmetric Action is described in section 6. A summary and discussion of our results is presented in section 7.

2 The $\mathcal{N} = 2$ supersymmetric model

As discussed in the introduction, vortex solutions in a model with two U(1) gauge fields A_μ and C_μ , each one coupled to complex scalar fields s and t respectively and a gauge kinetic coupling were constructed in [19]. Although the model is defined in $d = 3 + 1$ space-time dimensions, the proposed vortex ansatz corresponds to axially symmetric configurations that are independent of one of the three spatial directions and hence the equations of motion that they obey can be obtained from a $2 + 1$ dimensional action. Indeed, magnetic vortex configurations in the Abelian Higgs model were originally discussed [22] as static, z -independent classical solutions of the Abelian Higgs model in $3 + 1$ dimensions Minkowski space. Being z -independent, they can be also taken as static solutions in $2 + 1$ dimensions where the supersymmetric extension can be more easily formulated. We shall take this point of view. Hence, static magnetic vortices instead of being “tubes” of quantized magnetic flux as in the $3 + 1$ case, have to be considered, in $2 + 1$ dimensions, as disks defined on the (x, y) plane. The magnetic field, instead of being a (pseudo)vector becomes a (pseudo)scalar concentrated on the disk with the Higgs field being approximately zero inside the disk and taking its vacuum expectation value outside it.

The $2 + 1$ action governing the dynamics of the model discussed in [19] can be written in the form,

$$S = \int d^3x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} s^\dagger \hat{\square} s - V_1(s) - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} - \frac{1}{2} t^\dagger \tilde{\square} t - V_2(t) + \frac{\xi}{2} F_{\mu\nu} G^{\mu\nu} \right), \quad (2.1)$$

where $d^3x = dt dx dy = dt r dr d\varphi$,

$$\begin{aligned} \hat{\square} &= (\partial_\mu - ieA_\mu)(\partial^\mu - ieA^\mu), & \tilde{\square} &= (\partial_\mu - igC_\mu)(\partial^\mu - igC^\mu), \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, & G_{\mu\nu} &= \partial_\mu C_\nu - \partial_\nu C_\mu, \end{aligned} \quad (2.2)$$

and the symmetry breaking potentials are,

$$V_1(s) = \frac{a}{4} (|s|^2 - s_0^2)^2, \quad V_2(t) = \frac{b}{4} (|t|^2 - t_0^2)^2. \quad (2.3)$$

The second order equations of motion derived from this Action in eq. (2.1) reveal a very rich structure of vortex solutions and the dependence on the parameters of the theory shows a large variety of phenomena that could be of interest in connection to the problems described in the introduction. Solutions have been found numerically, by solving the second order equations of motion, since it was not obvious how to find Bogomol’nyi equations for this system, due to the gauge-mixing term.

As it is well known the existence of Bogomol'nyi equations is closely related to the existence of a $\mathcal{N} = 2$ extensions of purely bosonic models exhibiting kinks, vortices or monopole solutions [21]–[26]. In general, such extensions are only possible for very particular symmetry breaking potentials (or even for vanishing potential, as it is the case for models with monopole or dyon solutions). Indeed, the form of these potentials is a requirement for supersymmetry to hold. Then, a practical way to find models accepting Bogomol'nyi equations is to formulate the problem in superspace, without an explicit potential, and let the supersymmetry algebra guide us towards the correct form for the action, that will have its coupling constants in the right ratios.

2.1 The supersymmetric action

Three-dimensional $N = 2$ supersymmetry can be obtained by dimensional reduction from the equivalent $N = 1$ four-dimensional theory. In practice, the supersymmetry representations are tractable by themselves, though we will keep the connection with the higher dimensional theory in mind during the process.

Let us first set up the technical tools needed to construct the SUSY extension of the model in eq. (2.1). In order to enforce two copies of supersymmetry one needs two sets of Grassmann variables $\theta^\alpha, \bar{\theta}^\alpha$. Note that there is only one spinor representation in three-dimensions. We work with the $(+ - -)$ signature, contract indices using $\epsilon_{\alpha\beta}$, and choose the following γ -matrices,

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad (2.4)$$

so that $\gamma^\mu \gamma^\nu = \eta^{\mu\nu} + i\epsilon^{\mu\nu\rho} \gamma_\rho$. We will abuse the bar notation: over spinorial coordinates, this refers to two independent quantities, but if λ is a (complexified) fermion field, then $\bar{\lambda} = \gamma^0 \lambda^\dagger$ is not independent. We write the super-derivatives analogously to those in the $d = 3 + 1$ dimensional case

$$\mathcal{D}_\alpha = \partial_\alpha + i(\gamma^\mu \bar{\theta})_\alpha \partial_\mu, \quad \bar{\mathcal{D}}_\alpha = \bar{\partial}_\alpha + i(\theta \gamma^\mu)_\alpha \partial_\mu, \quad \{\mathcal{D}_\alpha, \bar{\mathcal{D}}_\beta\} = 2i\gamma_{\alpha\beta}^\mu \partial_\mu. \quad (2.5)$$

We also use the standard chiral hypermultiplet representation Φ for the matter fields. It contains one complex scalar s and one full Dirac spinor i.e. two independent Majorana spinors written as one complex spinor ψ , along with a complex auxiliary F-term. This auxiliary field will not play any relevant role, since as we shall see, our bosonic theory is realised without a superpotential. We define these individual components by the action of the super-derivatives on the field, evaluated at $\theta = \bar{\theta} = 0$; we denote the evaluation with $|$. Hence,

$$\Phi| = s, \quad \mathcal{D}_\alpha \Phi| = \bar{\psi}_\alpha, \quad \mathcal{D}_\alpha \mathcal{D}_\beta \Phi| = \gamma_{\alpha\beta}^\mu \partial_\mu s + \epsilon_{\alpha\beta} F, \quad \mathcal{D}_\alpha \mathcal{D}^\beta \bar{\mathcal{D}}_\beta \Phi| = (\not{\partial} \bar{\psi})_\alpha^\dagger \quad (2.6)$$

and

$$\Phi^\dagger| = s^\dagger, \quad \bar{\mathcal{D}}_\alpha \Phi^\dagger| = \psi_\alpha, \quad \bar{\mathcal{D}}_\alpha \bar{\mathcal{D}}_\beta \Phi^\dagger| = (\gamma_{\alpha\beta}^\mu \partial_\mu s + \epsilon_{\alpha\beta} F)^\dagger, \quad \mathcal{D}_\alpha \bar{\mathcal{D}}^\alpha \bar{\mathcal{D}}_\beta \Phi| = (\not{\partial} \psi)_\alpha. \quad (2.7)$$

The other matter superfield Ψ is treated identically and contains the scalars t , a Dirac fermion σ and an auxiliary field G .

The vector multiplet U possesses one real scalar M , two Majorana fermions written as one complex fermion $\lambda = \chi + i\rho$ and one gauge field A_μ , along with several auxiliary fields, all but one of which we can choose to ignore in the Wess-Zumino gauge. The only auxiliary field we cannot gauge away in this multiplet, we will call it D . Thus we define,

$$\mathcal{D}_\alpha \bar{\mathcal{D}}_\beta U| = \epsilon_{\alpha\beta} M + \gamma_{\alpha\beta}^\mu A_\mu, \quad \mathcal{D}^2 \bar{\mathcal{D}}^2 U| = D, \quad \mathcal{D}^2 \bar{\mathcal{D}}_\alpha U| = \bar{\chi}_\alpha, \quad \mathcal{D}_\alpha \bar{\mathcal{D}}^2 U| = \rho_\alpha. \quad (2.8)$$

The standard gauge-invariant curvature tensor W_α and its conjugate can be defined to generate the canonical kinetic term. In three dimension there exists an extra representation called the linear multiplet $\Sigma = \bar{\mathcal{D}}\mathcal{D}U$ which is real and obeys $\mathcal{D}^2 \Sigma = \bar{\mathcal{D}}^2 \Sigma = 0$. This field proves to be more convenient for component definitions as it contains all the degrees of freedom at once. Thus, using our previous results we define,

$$\Sigma| = M, \quad \mathcal{D}_\alpha \bar{\mathcal{D}}_\beta \Sigma| = \epsilon_{\alpha\beta} D + i\gamma_{\alpha\beta}^\mu (i\epsilon_{\mu\nu\rho} \partial^\nu A^\rho + \partial_\mu M), \quad (2.9)$$

$$\mathcal{D}_\alpha \Sigma| = \bar{\chi}_\alpha, \quad \bar{\mathcal{D}}_\alpha \Sigma| = \rho_\alpha, \quad \mathcal{D}_\alpha \bar{\mathcal{D}}^2 \Sigma| = i\gamma_{\alpha\beta}^\mu \partial_\mu \chi_\beta, \quad \mathcal{D}^2 \bar{\mathcal{D}}_\alpha \Sigma| = i\gamma_{\beta\alpha}^\mu \partial_\mu \bar{\rho}_\beta. \quad (2.10)$$

A second multiplet to describe the ‘hidden’ sector, the analogous to Σ , will be called Υ with bosonic fields C_μ , N , auxiliary field d and fermions $\tau = \zeta + i\omega$. With these definitions, we can now write the superspace action for our model;

$$S_{\mathcal{N}=2} = \int d^3x d^2\theta d^2\bar{\theta} \left(\frac{1}{4} \Sigma \Sigma + \frac{1}{4} \Phi^\dagger e^{-ieU} \Phi + \frac{1}{4} \Upsilon \Upsilon + \frac{1}{4} \Psi^\dagger e^{-igV} \Psi - \frac{\xi}{2} \Sigma \Upsilon + \frac{ies_0^2}{2} U + \frac{igt_0^2}{2} V \right). \quad (2.11)$$

The final two terms in the action are Fayet-Iliopoulos terms introduced to achieve the phenomenologically required spontaneous gauge symmetry breaking. Noting that for any field Δ

$$\int d^2\theta d^2\bar{\theta} \Delta \doteq \mathcal{D}^2 \bar{\mathcal{D}}^2 \Delta|, \quad (2.12)$$

it is then straightforward to use the previous relations, expand the superfields in components and obtain the SUSY-completed action for the pair of coupled Abelian Higgs system. We assume that the spinors, which all come in pairs, have been complexified according to $\lambda = \chi + i\rho$ and $\tau = \zeta + i\omega$. It is convenient to write the complete action in the form,

$$S_{\mathcal{N}=2} = S_1(A, s, M, \psi, \lambda) + S_2(C, t, N, \sigma, \tau) - \xi S_{\text{int}}(A, M, C, N, \lambda, \tau, d, D), \quad (2.13)$$

with

$$S_1 = \int d^3x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} s^\dagger \hat{\square} s + \frac{1}{2} D^2 - \frac{ie}{2} (|s|^2 - s_0^2) D - \frac{1}{2} M \square M - \frac{1}{4} M^2 |s|^2 + \frac{i}{2} \bar{\psi} \not{D}_A \psi + \frac{i}{2} \bar{\lambda} \not{\partial} \lambda - \frac{1}{2} M \bar{\psi} \psi - \frac{e}{2} (\bar{\psi} \lambda s + s^\dagger \bar{\lambda} \psi) \right). \quad (2.14)$$

The action S_1 , describing the ‘visible’ sector, is invariant under the following transformations with infinitesimal anticommuting complex parameters η ,

$$\begin{aligned} \delta s &= \bar{\eta}\psi, & \delta\psi &= -i\gamma_\mu\eta D_A^\mu s - \eta Ms, & \delta M &= \bar{\eta}\lambda - \bar{\lambda}\eta, & \delta A_\mu &= -i\bar{\eta}\gamma_\mu\lambda + i\bar{\lambda}\gamma_\mu\eta \\ \delta D &= \partial^\mu(\eta\gamma_\mu\bar{\lambda} - \bar{\eta}\gamma_\mu\lambda), & \delta\lambda &= -\epsilon^{\mu\nu\rho}\partial_\mu A_\nu\gamma_\rho\eta - i\not{\partial}M\eta - iD\eta. \end{aligned} \quad (2.15)$$

The second (‘dark’) sector has a similar action,

$$\begin{aligned} S_2 &= \int d^3x \left(-\frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{2}t^\dagger\bar{\square}t + \frac{1}{2}d^2 - \frac{ig}{2}(|t|^2 - t_0^2)d \right. \\ &\quad \left. - \frac{1}{2}N\square N - \frac{1}{4}N^2|t|^2 + \frac{i}{2}\bar{\sigma}\not{D}_C\sigma + \frac{i}{2}\bar{\tau}\not{\partial}\tau - \frac{1}{2}N\bar{\sigma}\sigma - \frac{g}{2}(\bar{\sigma}\tau t + t^\dagger\bar{\tau}\sigma) \right). \end{aligned} \quad (2.16)$$

The action S_2 is invariant under the following transformations,

$$\begin{aligned} \delta t &= \bar{\eta}\sigma, & \delta\sigma &= -i\gamma_\mu\eta D_C^\mu t - \eta Nt, & \delta N &= \bar{\eta}\tau - \bar{\tau}\eta, & \delta C_\mu &= -i\bar{\eta}\gamma_\mu\tau + i\bar{\tau}\gamma_\mu\eta \\ \delta d &= \partial^\mu(\eta\gamma_\mu\bar{\tau} - \bar{\eta}\gamma_\mu\tau), & \delta\tau &= -\epsilon^{\mu\nu\rho}\partial_\mu C_\nu\gamma_\rho\eta - i\not{\partial}N\eta - id\eta. \end{aligned} \quad (2.17)$$

The term coupling the two visible and dark sectors, which is invariant under both sets of transformations reads

$$S_{\text{int}} = \int d^3x \left(-\frac{1}{2}F_{\mu\nu}G^{\mu\nu} - \frac{1}{2}(M\square N + N\square M) + \frac{i}{2}(\bar{\lambda}\not{\partial}\tau + \bar{\tau}\not{\partial}\lambda) + dD \right). \quad (2.18)$$

Let us discuss the form of the scalar potential derived from the auxiliary fields.

2.2 The scalar potential

In order to obtain the symmetry breaking potential we have to solve the equations of motion for auxiliary fields D and d whose contribution will be collected in L_{Dd} ,

$$L_{Dd} = \frac{1}{2}D^2 + \frac{1}{2}d^2 - \xi dD - \frac{ie}{2}(|s|^2 - s_0^2)D - \frac{ig}{2}(|t|^2 - t_0^2)d, \quad (2.19)$$

or

$$L_{dD} = \frac{1}{2} \begin{pmatrix} D & d \end{pmatrix} \begin{pmatrix} 1 & -\xi \\ -\xi & 1 \end{pmatrix} \begin{pmatrix} D \\ d \end{pmatrix} - \begin{pmatrix} D & d \end{pmatrix} \begin{pmatrix} \frac{ie}{2}(|s|^2 - s_0^2) \\ \frac{ig}{2}(|t|^2 - t_0^2) \end{pmatrix}. \quad (2.20)$$

The extrema of this quadratic system is given by,

$$\begin{pmatrix} D \\ d \end{pmatrix} = \frac{1}{1 - \xi^2} \begin{pmatrix} 1 & \xi \\ \xi & 1 \end{pmatrix} \begin{pmatrix} \frac{ie}{2}(|s|^2 - s_0^2) \\ \frac{ig}{2}(|t|^2 - t_0^2) \end{pmatrix}, \quad (2.21)$$

which gives,

$$D = \frac{i}{2} \frac{1}{1 - \xi^2} \left(e(|s|^2 - s_0^2) + \xi g(|t|^2 - t_0^2) \right) \quad (2.22)$$

$$d = \frac{i}{2} \frac{1}{1 - \xi^2} \left(e\xi(|s|^2 - s_0^2) + g(|t|^2 - t_0^2) \right). \quad (2.23)$$

Substituting this expression in eq. (2.20) one gets,

$$L_{Dd} = -\frac{1}{2(1-\xi^2)} \left(\frac{ie}{2}(|s|^2 - s_0^2) \quad \frac{ig}{2}(|t|^2 - t_0^2) \right) \begin{pmatrix} 1 & \xi \\ \xi & 1 \end{pmatrix} \begin{pmatrix} \frac{ie}{2}(|s|^2 - s_0^2) \\ \frac{ig}{2}(|t|^2 - t_0^2) \end{pmatrix}. \quad (2.24)$$

So that finally the scalar potential takes the form

$$V[s, t] = \frac{1}{2(1-\xi^2)} \left(\frac{e^2}{4}(|s|^2 - s_0^2)^2 + \frac{g^2}{4}(|t|^2 - t_0^2)^2 + \frac{eg\xi}{2}(|s|^2 - s_0^2)(|t|^2 - t_0^2) \right). \quad (2.25)$$

Clearly this reduces to the usual, decoupled form in the case $\xi = 0$. To avoid singularities, the parameter ξ should be constrained so that $|\xi| < 1$. In fact, as discussed in [19], the existence of solutions with the appropriate asymptotic boundary conditions imposes such constraint.

The equations for the extrema of the potential read

$$\begin{aligned} e^2|s|(|s|^2 - s_0^2) + ge\xi|s|(|t|^2 - t_0^2) &= 0 \\ g^2|t|(|t|^2 - t_0^2) + eg\xi|t|(|s|^2 - s_0^2) &= 0 \end{aligned}$$

and the Hessian matrix is given by

$$\mathcal{H} = \frac{1}{2(1-\xi^2)} \begin{pmatrix} 3e^2|s|^2 + e^2s_0^2 + eg\xi(|t|^2 - t_0^2) & 2eg\xi|s||t| \\ 2eg\xi|s||t| & 3g^2|t|^2 + g^2t_0^2 + eg\xi(|s|^2 - s_0^2) \end{pmatrix} \quad (2.26)$$

There are 4 particular types of critical points of interest:

- $|s| = |t| = 0$: maximum, the vacuum is unstable.
- $|s| = 0, |t| = \sqrt{t_0^2 + \frac{e\xi}{g}s_0^2}$: saddle point
- $|s| = \sqrt{s_0^2 + \frac{g\xi}{e}t_0^2}, |t| = 0$: saddle point²
- $|s| = s_0, |t| = t_0$: this is the true minimum.

This verifies that we have perturbed each of the Abelian Higgs Models. Let us now turn to the main point of this procedure, the BPS equations.

3 Bogomol'nyi equations

In this section we will derive BPS equations for the system described in eq. (2.1). We will follow the well-known procedure, basically imposing that a purely bosonic configuration preserves part of the SUSY of the action in eq. (2.13).

²These saddle points can disappear if ξ is negative, and if the VEVs and coupling constants take such values as to make the inside of these square roots negative.

3.1 BPS states and equations

Starting from the $\mathcal{N} = 2$ supersymmetric action in eq. (2.13) we are interested in finding a purely bosonic action in which gauge fields and Higgs scalars in the visible and hidden sectors are coupled in such a way that first order BPS equations do exist. This will be achieved by enforcing that only the bosonic part of the $\mathcal{N} = 2$ supersymmetric action eq. (2.13) subsists. Following this procedure one ends with extra adjoint (i.e. ungauged) scalars for each gauge group, which we also require to be zero.

The rationale behind this procedure is well-known: we are free to impose that physical states have only the degrees of freedom we desire (gauge particles and squarks) and not break supersymmetry completely by imposing that the supersymmetric variations of each vanishing field is identically zero. Such states are called BPS states, because they saturate the Bogomol'nyi lower bound for the total energy of the system. Recalling the supersymmetric variations previously established, and imposing that only our physical fields appear, we obtain the following set of equations, for an arbitrary infinitesimal spinor η by demanding that the variations of the fermion fields vanish;

$$\begin{aligned} -i\gamma_\mu\eta D_A^\mu s &= 0, & -\epsilon^{\mu\nu\rho}\partial_\mu A_\nu\gamma_\rho\eta - iD\eta &= 0, \\ -i\gamma_\mu\eta D_C^\mu t &= 0, & -\epsilon^{\mu\nu\rho}\partial_\mu C_\nu\gamma_\rho\eta - id\eta &= 0. \end{aligned} \tag{3.1}$$

Furthermore, to write the equations leading to magnetic vortex solutions, we shall impose time-independence of our solutions and use the gauge choice $A_0 = C_0 = 0$. Rewriting the above we get,

$$\begin{aligned} (-i\gamma_1\eta D_A^1 - i\gamma_2\eta D_A^2)s &= 0, & -\epsilon^{0ij}\partial_i A_j\gamma_0\eta - iD\eta &= 0 \\ (-i\gamma_1\eta D_C^1 - i\gamma_2\eta D_C^2)t &= 0, & -\epsilon^{0ij}\partial_i C_j\gamma_0\eta - id\eta &= 0 \end{aligned} \tag{3.2}$$

Then, we multiply the scalar equations by γ^1 and notice that in all cases the equations are proportional to the identity or γ^0 times the arbitrary spinor η . The resulting equations, acting on each of the spinor components, differ only by a sign change, so clearly they cannot be satisfied both at the same time but are valid for a definite sign choice in all cases. This is why precisely half of supersymmetry is broken (either the symmetry associated with η_+ or η_- must be selected). After using the eqs. (2.22)–(2.23) for D and d , the Bogomol'nyi equations read,

$$\begin{aligned} i\epsilon_{ij}D_A^i s &= \pm(D_j^A s)^*, & \epsilon_{ij}\partial_i A_j &= \pm\frac{1}{2}\frac{1}{1-\xi^2}\{e(|s|^2 - s_0^2) + g\xi(|t|^2 - t_0^2)\} \\ i\epsilon_{ij}D_C^i t &= \pm(D_j^C t)^*, & \epsilon_{ij}\partial_i C_j &= \pm\frac{1}{2}\frac{1}{1-\xi^2}\{g(|t|^2 - t_0^2) + e\xi(|s|^2 - s_0^2)\} \end{aligned} \tag{3.3}$$

3.2 The BPS bound for the energy

As a consistency check, we shall re-derive the self-dual equations (3.3) following the Bogomol'nyi approach [20]. This consist in writing the energy as a manifestly positive quantity plus a topological term. In this way a bound for the energy can be obtained. A set of first order equations precisely saturate this bound.

Once the extra scalars M and N , as well as the fermion fields, are put to zero, and the auxiliary fields D and d are written in terms of the dynamical bosonic fields, the Lagrangian associated to the action in eq. (2.13) reads,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{\xi}{2}F_{\mu\nu}G^{\mu\nu} + \frac{1}{2}|D_\mu(A)s|^2 + \frac{1}{2}|D_\mu(C)t|^2 - V. \quad (3.4)$$

Where

$$V = \frac{1}{2(1-\xi^2)} \left(\frac{e^2}{4}(|s|^2 - s_0^2)^2 + \frac{g^2}{4}(|t|^2 - t_0^2)^2 + \frac{eg\xi}{2}(|s|^2 - s_0^2)(|t|^2 - t_0^2) \right). \quad (3.5)$$

We are looking for static vortex-like classical solutions to the equation of motion in the gauge $A_0 = C_0 = 0$, which have quantized magnetic fluxes associated to A_i and C_i

$$\Phi_A = \oint A_i dx^i = \frac{2\pi n}{e}, \quad \Phi_C = \oint C_i dx^i = \frac{2\pi k}{g}, \quad n, k \in \mathbf{Z}. \quad (3.6)$$

To this end, it is convenient to introduce dimensionless variables,

$$x_i \rightarrow x_i/es_0, \quad A_i \rightarrow A_i s_0, \quad s \rightarrow s s_0, \quad C_i \rightarrow C_i s_0, \quad t \rightarrow t s_0 \quad (3.7)$$

and the coupling constants and gauge field masses ratios,

$$e_r \equiv g/e, \quad \mu^2 \equiv e_r^2 t_0^2 / s_0^2. \quad (3.8)$$

In terms of these fields, the total energy of the system is

$$\frac{E}{\ell} = s_0^2 \int d^2x \left\{ \frac{B_A^2}{2} + \frac{B_C^2}{2} + \frac{1}{2}|\partial_i s - iA_i s|^2 + \frac{1}{2}|\partial_i t - ie_r C_i t|^2 - \xi B_A B_C + V(|s|) + V(|t|) + V_{\text{int}} \right\}, \quad (3.9)$$

with $\ell = 1/(es_0)$ and magnetic fields B_A and B_C defined as,

$$B_A = \epsilon_{ij}\partial_i A_j = F_{12}, \quad B_C = \epsilon_{ij}\partial_i C_j = G_{12}. \quad (3.10)$$

Concerning the potentials, we have

$$V(|s|) = \frac{1}{8(1-\xi^2)}(|s|^2 - 1)^2, \quad V(|t|) = \frac{e_r^2}{8(1-\xi^2)} \left(|t|^2 - \left(\frac{\mu}{e_r} \right)^2 \right)^2, \quad (3.11)$$

and

$$V_{\text{int}} = \frac{e_r \xi}{4(1-\xi^2)}(|s|^2 - 1) \left(|t|^2 - \left(\frac{\mu}{e_r} \right)^2 \right). \quad (3.12)$$

To study the equations of motion, let us introduce a cylindrically symmetric ansatz for the fields, in terms of radial functions as

$$A_\varphi = \frac{\alpha(r)}{r}, \quad C_\varphi = \frac{\gamma(r)}{e_r r}, \quad s = \rho(r)e^{i\varphi}, \quad t = p(r)e^{i\varphi}. \quad (3.13)$$

The energy density in terms of the radial functions takes the form

$$\begin{aligned} \mathcal{E} = & \frac{1}{2r^2} \left(\frac{d\alpha}{dr} \right)^2 + \frac{1}{2r^2 e_r^2} \left(\frac{d\gamma}{dr} \right)^2 + \frac{1}{2} \left(\left(\frac{d\rho}{dr} \right)^2 + (\alpha - 1)^2 \frac{\rho^2}{r^2} \right) \\ & + \frac{1}{2} \left(\left(\frac{dp}{dr} \right)^2 + (\gamma - 1)^2 \frac{p^2}{r^2} \right) - \frac{\xi}{e_r} \frac{d\alpha}{dr} \frac{d\gamma}{dr} + V(\rho) + V(p) + V_{\text{int}} \end{aligned} \quad (3.14)$$

It is clear from the equation above that if the parameter ξ/e_r gets bigger, the magnetic energy diminishes.

Concerning the second order radial equation of motion they read,

$$r \frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} \left(\alpha + \frac{\xi}{e_r} \gamma \right) \right] + (1 - \alpha) \rho^2 = 0, \quad (3.15)$$

$$r \frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} (\gamma + \xi e_r \alpha) \right] + e_r^2 (1 - \gamma) p^2 = 0, \quad (3.16)$$

$$\frac{1}{r} \frac{d}{dr} \left[r \frac{d}{dr} \rho \right] - \frac{1}{r^2} (\alpha - 1)^2 \rho - \frac{1}{2(1 - \xi^2)} \left(\rho^2 - 1 + e_r \xi \left(p^2 - \frac{\mu^2}{e_r^2} \right) \right) \rho = 0, \quad (3.17)$$

$$\frac{1}{r} \frac{d}{dr} \left[r \frac{d}{dr} p \right] - \frac{1}{r^2} (\gamma - 1)^2 p - \frac{e_r^2}{2(1 - \xi^2)} \left(p^2 - \frac{\mu^2}{e_r^2} + \frac{\xi}{e_r} (\rho^2 - 1) \right) p = 0. \quad (3.18)$$

Notice that in the limit case of $\xi = e_r = \mu = 1$, the kinetic term for the combination of gauge fields $\alpha - \gamma$ decouples.

In view of the well-known connection between BPS states and the Bogomol'nyi bound to the energy [23]–[24], we know that the solution to eqs. (3.3) also solve the equations of motion in eqs. (3.15)–(3.18). Indeed, the energy in eq. (3.9) can be rewritten as a manifestly positive quantity, by “completing the square”, which in this case is not just a square but a positive definite quadratic form.

$$\begin{aligned} E = & \int d^2x \left(\frac{1}{2} \left(B_A \mp \frac{1}{2(1 - \xi^2)} \left(e(|s|^2 - s_0^2) + \xi g(|t|^2 - t_0^2) \right) \right) \right)^2 \\ & + \frac{1}{2} \left(B_C \mp \frac{1}{2(1 - \xi^2)} \left(g(|t|^2 - t_0^2) + \xi e(|s|^2 - s_0^2) \right) \right)^2 \\ & - \xi \left(B_A \mp \frac{1}{2(1 - \xi^2)} \left(e(|s|^2 - s_0^2) + \xi g(|t|^2 - t_0^2) \right) \right) \\ & \quad \times \left(B_C \mp \frac{1}{2(1 - \xi^2)} \left(g(|t|^2 - t_0^2) + \xi e(|s|^2 - s_0^2) \right) \right) \\ & + \left(\frac{1}{2} |\varepsilon_{ij} D_j [A] s_a \mp \epsilon_{ab} D_i [A] s_b|^2 + \frac{1}{2} |\varepsilon_{ij} D_j [C] t_a \mp \epsilon_{ab} D_i [C] t_b|^2 \pm \partial^i \mathcal{J}_i \right). \end{aligned} \quad (3.19)$$

Here we have written the real and imaginary components of the scalar fields as s_a with $a = 1, 2$ respectively and defined covariant derivatives as

$$D_i s_a = \partial_i s_a + \epsilon_{ab} A_i s_b, \quad (3.20)$$

and analogously for the hidden scalar field t_a . All but the last term in this energy are part of a positive-definite quadratic form, so that this part of the energy is always positive. Concerning the last term — generated to complete the form — the current \mathcal{J}_i is given by

$$\mathcal{J}_i = \varepsilon_{ij} \left(\varepsilon_{ab} (s_a D_j [A] s_b + A_j) \right) + \varepsilon_{ij} \left(\varepsilon_{ab} \left(t_a D_j [C] t_b + \frac{1}{e_r} C_j \right) \right). \quad (3.21)$$

Using Stoke's theorem and using the fact that covariant derivatives vanish at infinity (since we impose finite energy for our solutions) one ends with the Bogomol'nyi bound for the energy which written in terms of the original (unscaled) fields reads

$$E \geq es_0^2 |\Phi_A| + et_0^2 |\Phi_C| = s_0^2 2\pi |n| + t_0^2 2\pi |k| \quad (3.22)$$

The bound is attained when each one of the squares in eq. (3.19) vanish, this leading precisely to the already obtained equations (3.3), that written in terms of the Ansatz read

$$\frac{d\rho}{dr} = (-1 + \alpha) \frac{\rho}{r}, \quad (3.23)$$

$$\frac{dp}{dr} = (-1 + \gamma) \frac{p}{r}, \quad (3.24)$$

$$\frac{1}{r} \frac{d\alpha}{dr} = \pm \frac{1}{2(1 - \xi^2)} \left[(\rho^2 - 1) + e_r \xi \left(p^2 - \frac{\mu^2}{e_r^2} \right) \right], \quad (3.25)$$

$$\frac{1}{r} \frac{d\gamma}{dr} = \pm \frac{1}{2(1 - \xi^2)} \left[e_r \left(p^2 - \frac{\mu^2}{e_r^2} \right) + \xi (\rho^2 - 1^2) \right]. \quad (3.26)$$

Note that if $\xi = \pm 1$ in eq. (3.19) the purely positive part of the energy degenerates and can be factorised again into a simpler expression.

In summary, we have obtained the BPS equations for our system in eq. (2.1). We will now study how its topological charge can be re-obtained using a different approach, based on the SUSY algebra.

4 Supercharges

The $\mathcal{N} = 2$ action of eq. (2.13) is invariant under two super-transformations, detailed in eqs. (2.15) and (2.17). The fermionic Noether charge associated with these invariances is given by,

$$\bar{Q}_\eta = \sum_{\zeta \in \text{fermions}} \frac{\delta S}{\delta(\partial_0 \zeta)} \delta_\eta \zeta. \quad (4.1)$$

This gives the following expressions

$$\begin{aligned} \bar{Q} = \int d^2x & \left((\bar{\lambda} - \xi \bar{\tau}) \gamma^0 \left(-\frac{1}{2} \epsilon^{\mu\nu\rho} F_{\mu\nu} \gamma_\rho - i \not{\partial} M - i D \right) + (\bar{\tau} - \xi \bar{\lambda}) \gamma^0 \left(-\frac{1}{2} \epsilon^{\mu\nu\rho} G_{\mu\nu} \gamma_\rho - i \not{\partial} N - i d \right) \right. \\ & \left. + \bar{\sigma} \gamma^0 \left(-i(\not{\partial} - ig\not{C})t - \frac{1}{2} N t \right) + \bar{\psi} \gamma^0 \left(-i(\not{\partial} - ie\not{A})s - \frac{1}{2} M s \right) \right). \end{aligned} \quad (4.2)$$

$$\begin{aligned} Q = \int d^2x & \left(\left(-\frac{1}{2} \epsilon^{\mu\nu\rho} (F_{\mu\nu} - \xi G_{\mu\nu}) \gamma_\rho + i \not{\partial} (M - \xi N) + i(D - \xi d)^* \right) \gamma^0 \lambda \right. \\ & + \left(-\frac{1}{2} \epsilon^{\mu\nu\rho} (G_{\mu\nu} - \xi F_{\mu\nu}) \gamma_\rho + i \not{\partial} (N - \xi M) + i(d - \xi D)^* \right) \gamma^0 \tau \\ & \left. + \left(i(\not{\partial} + ig\not{C})t^* - \frac{g}{2} N t^* \right) \gamma^0 \sigma + \left(i(\not{\partial} + ie\not{A})s^* - \frac{e}{2} M s^* \right) \gamma^0 \psi \right) \end{aligned} \quad (4.3)$$

These charges have been rewritten in terms of the fermionic fields (in Q) and their conjugate momenta (in \bar{Q}) so as to more easily impose canonical anti-commutation relations later on. We have not yet substituted for the on-shell value of D and d given in eq. (2.21).

We can now rederive the Bogomol'nyi bound on the energy of the system, and its saturation by self-dual equations from the supercharge algebra. Indeed, as it is well known [21]–[26] that in the supersymmetry context the Bogomol'nyi equations imply that the total energy of the system is bounded below by the central charge of the theory which is proportional to the topological charge associated to the solutions of the self-dual equations (in our case the number of vortex flux units of both sectors).

The Supersymmetry algebra dictates that, in the rest frame

$$\{Q_\alpha, \bar{Q}_\beta\} = \gamma_{\alpha\beta}^0 E + \mathbb{1}_{\alpha\beta} T, \quad (4.4)$$

here E is the total energy of the system and T is the central charge. Squaring and tracing over this equation it is easy to get,

$$E \geq |T|, \quad (4.5)$$

which is the promised bound. Now, using the explicit form of the supercharges in eqs. (4.2)–(4.3), we can calculate explicitly E and T . Since we will eventually only keep bosonic terms, we need to impose that the fermions and their canonical conjugate obey usual Hamiltonian mechanics commutation relations. Because of the presence of the mixing term, the conjugate momentum for the gauginos is not trivial. Indeed, we find that

$$\frac{\delta S}{\delta \partial_0 \lambda} = (\bar{\lambda} - \xi \bar{\tau}) \gamma^0, \quad \frac{\delta S}{\delta \partial_0 \tau} = (\bar{\tau} - \xi \bar{\lambda}) \gamma^0. \quad (4.6)$$

We therefore impose that

$$\{\lambda_\alpha(x), (\bar{\lambda}(y) \gamma^0 - \xi \bar{\tau}(y) \gamma^0)_\beta\} = \{\tau_\alpha(x), (\bar{\tau}(y) \gamma^0 - \xi \bar{\lambda}(y) \gamma^0)_\beta\} = \mathbb{1}_{\alpha\beta} \delta^{(3)}(x - y), \quad (4.7)$$

and put fermions to zero after calculating the anti-comutators. We also set the gauge scalars to zero and impose the standard assumptions for magnetic vortex solutions (the

gauge choice $A_0 = C_0 = 0$ and time-independence of the configuration). The resulting energy and central charge are,

$$\begin{aligned}
 E &= \int d^2x \left(\frac{1}{2} F_{ij} F^{ij} + \frac{1}{2} G_{ij} G^{ij} - \xi F^{ij} G_{ij} - D^2 - d^2 + \xi Dd + |D^A s|^2 + |D^C t|^2 \right) \\
 &= \int d^2x \left(e \frac{1}{2} F_{ij} F^{ij} + \frac{1}{2} G_{ij} G^{ij} - \xi F^{ij} G_{ij} + V(s, t) + |D^A s|^2 + |D^C t|^2 \right), \quad (4.8)
 \end{aligned}$$

$$T = i\epsilon^{ij} \int d^2x (F_{ij} D + G_{ij} d - \xi F_{ij} d - \xi G_{ij} D + (D_i^A s)(D_j^A s)^* + (D_i^C t)(D_j^C t)^*). \quad (4.9)$$

Inserting auxiliary fields D and d as given in eq. (2.21) we get

$$T = -\int d^2x \left(\epsilon^{ij} F_{ij} \frac{e}{2} (|s|^2 - s_0^2) + \epsilon^{ij} G_{ij} \frac{g}{2} (|t|^2 - t_0^2) + i\epsilon^{ij} (D_i^A s)(D_j^A s)^* + i\epsilon^{ij} (D_i^C t)(D_j^C t)^* \right) \quad (4.10)$$

which can be rewritten as

$$T = \int d^2x \partial_i (\pi_1^i + \pi_2^i), \quad (4.11)$$

with

$$\pi_1^i = \epsilon^{ij} \left(A_j \frac{e}{2} s_0^2 + i s^* D_j^A s \right), \quad \pi_2^i = \epsilon^{ij} \left(C_j \frac{g}{2} t_0^2 + i t^* D_j^C t \right). \quad (4.12)$$

Using Stoke's theorem we are left with a contour integral of these quantities over the circle at infinity. Since covariant derivatives should vanish on this contour, we get

$$T = e s_0^2 \oint A_i dx^i + g t_0^2 \oint C_i dx^i = s_0^2 2\pi |n| + t_0^2 2\pi |k|. \quad (4.13)$$

Then, using eq. (4.5) we obtain the same bound as in eq. (3.22). We used the more algorithmic procedure based on the SUSY algebra.

It should be noted that the interaction between vortices contributes no net central charge (i.e. T does not depend on ξ), only extra energy through the gauge quadratic term and the extra part of the scalar potential, as seen above. This is natural; T is a topological quantity. It cannot depend on smoothly-varying parameters.

5 Numerical solutions

In this section we present the vortex solutions to eqs. (3.23)–(3.26) obtained using an asymptotic shooting method [27]. We use the Euler-Lagrange radial equations (3.42)–(3.45) since, being second order, there are two integration constants per equation in contrast with just one in the first order Bogomol'nyi case. This gives more degrees of freedom for the method to act on, and bases itself on a system that suffers less unstable behaviours as we move through parameter space, justifying the use of these (a priori more difficult) equations.

Of course, we finally achieve complete agreement for the solutions of both first and second order systems. We plot the solution profiles and its dependence on the free parameters $[\xi, e_r, \mu]$ defined in eq. (3.6), showing that these parameters can be tuned to have different

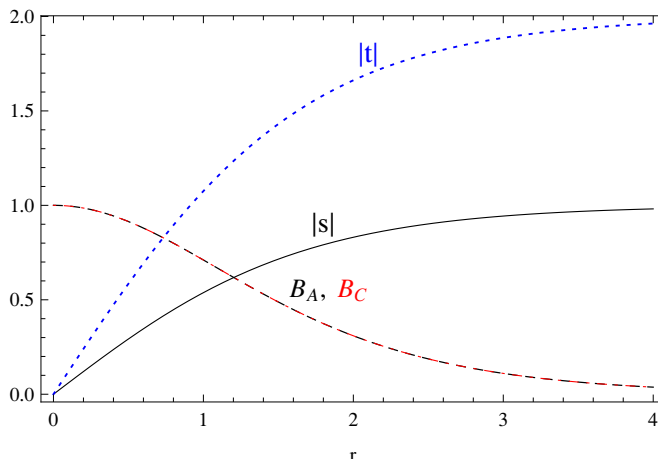


Figure 1. Scalar and magnetic field profiles for $\xi = 10^{-8}$, $e_r = 0.5$, $\mu = 1$.

profiles and widths for the hidden and visible strings. The idea is that this analysis could be useful to determine the best conditions and experimental framework to study the hidden sector, specially in connection with hidden dark strings interacting with particles of the Standard Model. We will keep the kinetic gauge mixing ξ in the region $\xi < 10^{-3}$ since larger values are experimentally ruled out. Concerning e_r , its value controls the vacuum expectation values of the visible and hidden scalar fields.

We started our analysis by considering the $\xi \rightarrow 0$ limit in which, as expected, the solutions correspond to those of two decoupled Abelian Higgs models, namely the usual Nielsen-Olesen vortices.

Then, by taking larger values of the mixing parameter we studied the deviation of vortex solutions (both of the Higgs fields s, t profiles and the magnetic. Below we present the most relevant features of the resulting solutions.

- *The decoupled case.* We first considered a small value of the kinetic mixing parameter, $\xi = 10^{-8}$, and *identical* values for the gauge couplings and vector masses in both sectors, $e_r = \mu = 1$. As advanced, the two sectors decouple showing each one Nielsen-Olesen vortex solutions.

Nonetheless, still in the very small mixing parameter regime, appreciable departures from the decoupled Nielsen-Olesen solutions were found if, for instance, the two gauge coupling constant are different, $e_r \neq 1$. In this case, the rescaled VEVs of the two Abelian Higgs are different, see eqs. (3.11)–(3.12), therefore showing different profiles. The fields profile for the case $e_r = 0.5$ is shown in figure 1. As can be seen from this figure, when e_r decreases (for fixed μ), the expectation value of the hidden Higgs field grows. Note that in this case the magnetic fields from both sectors remain identical.

We studied larger values $e_r > 1$ and the result above still holds (the rescaled potential minimum of the hidden scalar is smaller). The main feature to be retained from these results is that even in the very small ξ regime, the Higgs scalars in each sector

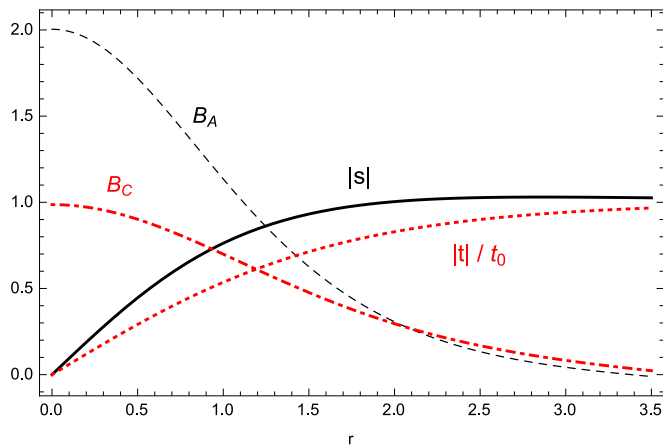


Figure 2. Field profiles for $\xi = 10^{-8}$ and $e_r = 10^{-8}$, $\mu = 1$. The hidden Higgs scalar t is rescaled by its VEV to fit in the figure.

detect the gauge mixing showing different profiles while the magnetic fields remain indistinguishable when the gauge field masses are identical, see figure 1.

It is important to note at this point that the actual parameter controlling the strength of the mixing in the kinetic sector is in fact given by the quotient ξ/e_r , as can be seen from equation (3.14). Then, even if in agreement with experimental constraints one considers very small ξ values, the ratio ξ/e_r can be made closer, or even bigger than one by considering the visible gauge coupling constant much bigger than the hidden sector one, this implying $e_r \ll 1$. Such possibility is shown in figure 2, in which $\xi = 10^{-8}$ and $e_r = 10^{-8}$. One can see that in that case the visible and hidden magnetic fields differ. Concerning scalars, the visible Higgs field profile remains similar to that in figure 1, while the hidden Higgs has been rescaled in the plot because of its much bigger VEV.

Concerning the case in which the masses of the gauge fields are different $\mu \neq 1$ (with the mixing parameter still very small, $\chi = 10^{-8}$) none of the fields profile is identical to their hidden counterpart, as can be seen in figure 3 for the case $\mu = 0.5$. Moreover, each sector exhibits profiles which coincide with the uncoupled ($\xi = 0$) case. Note that the choice corresponds to a hidden gauge field mass smaller than the visible one this implying that the exponential decay of the hidden magnetic field is slower, as can be seen in the figure. Now, since we are at the Bogomol’nyi point the mass of the hidden scalar is identical to the hidden gauge field mass, and hence the growth of the scalar field towards its VEV is slower.

- *“Strong” mixing regime.* As explained in section 2.2 consistency of asymptotic behaviors implies that $\xi < 1$. We then consider that the two sectors are strongly coupled for the gauge mixing parameter of the order $\xi \sim 0.5$ taking the ratio of gauge couplings $e_r = 0.5$. The profile of the fields is shown in figure 4 for the case $\xi \sim 0.5$ so that $\xi/e_r = 1$. One can see that the visible and hidden magnetic field profiles not only are different but the exhibit a crossover.

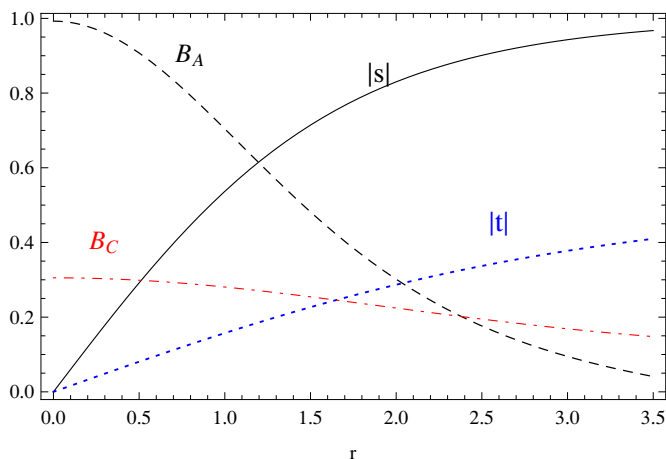


Figure 3. We have considered $\mu = 0.5$, $\xi = 10^{-8}$ and $e_r = 1$.

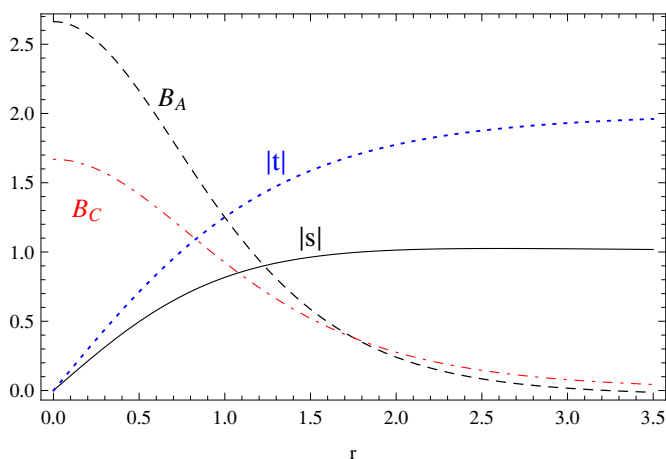


Figure 4. We have considered $\mu = 1$, $\xi = 0.5$ and $e_r = 0.5$.

6 Diagonalisation of the theory

6.1 The diagonal action

In his study of “millicharged-particles” [6], Holdom observed that when the gauge fields are mixed with a $F_{\mu\nu}G^{\mu\nu}$ term, one can perform a change of basis leading to an orthogonal diagonalisation of the kinetic terms. In this section we extend such procedure to the supersymmetric model extension that we are discussing.

We start from the $N = 2$ superspace action in eq. (2.11)

$$S_{N=2} = \int d^3x d^2\theta d^2\bar{\theta} \left\{ \frac{1}{4} \Sigma \Sigma + \frac{1}{4} \Phi^\dagger e^{-ieU} \Phi + \frac{ies_0^2}{2} U + \frac{1}{4} \Upsilon \Upsilon + \frac{1}{4} \Psi^\dagger e^{-igV} \Psi + \frac{igt_0^2}{2} V - \frac{\xi}{2} \Sigma \Upsilon \right\}, \quad (6.1)$$

and, as we did in section 2.2 for the scalar potential, we rewrite the supersymmetric Lagrangian for the coupled gauge fields sector as a quadratic form;

$$\begin{aligned} L_{\Sigma\Upsilon} &= \frac{1}{4} \begin{pmatrix} \Sigma & \Upsilon \end{pmatrix} \begin{pmatrix} 1 & -\xi \\ -\xi & 1 \end{pmatrix} \begin{pmatrix} \Sigma \\ \Upsilon \end{pmatrix} \\ &= \frac{1}{4} \begin{pmatrix} \Sigma & \Upsilon \end{pmatrix} \mathcal{M} \begin{pmatrix} \Sigma \\ \Upsilon \end{pmatrix}. \end{aligned} \tag{6.2}$$

The matrix \mathcal{M} in the second line of eq. (6.2) can be diagonalised by the following orthogonal matrix

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \mathcal{M} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1-\xi & 0 \\ 0 & 1+\xi \end{pmatrix}. \tag{6.3}$$

Hence we define new decoupled gauge multiplets,

$$\begin{pmatrix} \tilde{U} \\ \tilde{V} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix}. \tag{6.4}$$

Note that this change of basis is its own inverse and it induces a trivial Jacobian in the putative partition function of the model.

The new action, in terms of these gauge fields becomes,

$$\begin{aligned} S_{N=2} &= \int d^3x d^2\theta d^2\bar{\theta} \left(\frac{1-\xi}{4} \tilde{\Sigma} \tilde{\Sigma} + \frac{1}{2} \Phi^\dagger e^{-\frac{ie}{\sqrt{2}}(\tilde{U}+\tilde{V})} \Phi + \frac{ies_0^2}{2\sqrt{2}} (\tilde{U} + \tilde{V}) \right. \\ &\quad \left. + \frac{1+\xi}{4} \tilde{\Upsilon} \tilde{\Upsilon} + \frac{1}{4} \Psi^\dagger e^{-\frac{ig}{\sqrt{2}}(\tilde{U}-\tilde{V})} \Psi + \frac{igt_0^2}{2\sqrt{2}} (\tilde{U} - \tilde{V}) \right). \end{aligned} \tag{6.5}$$

If $\xi = \pm 1$ one of these eigenvalues would vanish, meaning one of the new gauge fields decouples. See below eq. (3.18) for an alternative view on the same effect. In order to canonically normalize the gauge kinetic terms we redefine superfields

$$\tilde{U} = \frac{1}{\sqrt{1-\xi}} \hat{U}, \quad \tilde{V} = \frac{1}{\sqrt{1+\xi}} \hat{V}, \tag{6.6}$$

and gauge charges,

$$e_1 = \frac{e}{\sqrt{2(1-\xi)}}, \quad e_2 = \frac{e}{\sqrt{2(1+\xi)}}, \quad g_1 = \frac{g}{\sqrt{2(1-\xi)}}, \quad g_2 = \frac{-g}{\sqrt{2(1+\xi)}}, \tag{6.7}$$

so that we end up with the following action,

$$\begin{aligned} S_{N=2} &= \int d^3x d^2\theta d^2\bar{\theta} \left(\frac{1}{4} \hat{\Sigma} \hat{\Sigma} + \frac{1}{4} \Phi^\dagger e^{(-ie_1 \hat{U} - ie_2 \hat{V})} \Phi + \frac{1}{2} (ie_1 s_0^2 + ig_1 t_0^2) \hat{U} \right. \\ &\quad \left. + \frac{1}{4} \hat{\Upsilon} \hat{\Upsilon} + \frac{1}{4} \Psi^\dagger e^{(-ig_1 \hat{U} - ig_2 \hat{V})} \Psi + \frac{1}{2} (ie_2 s_0^2 + ig_2 t_0^2) \hat{V} \right). \end{aligned} \tag{6.8}$$

This diagonal action is a more standard theory than the original one. Indeed, the matter sector is in a bi-fundamental representation of a $U(1) \times U(1)$ gauge group, with no direct

interaction between the gauge particles themselves. To recapitulate the new field content we have is,

$$\begin{aligned} \hat{A}_\mu &= \sqrt{\frac{1-\xi}{2}}(A_\mu + C_\mu), & \hat{M} &= \sqrt{\frac{1-\xi}{2}}(M+N), & \hat{\lambda} &= \sqrt{\frac{1-\xi}{2}}(\lambda+\tau), & \hat{D} &= \sqrt{\frac{1-\xi}{2}}(D+d), \\ \hat{C}_\mu &= \sqrt{\frac{1+\xi}{2}}(A_\mu - C_\mu), & \hat{M} &= \sqrt{\frac{1+\xi}{2}}(M-N), & \hat{\tau} &= \sqrt{\frac{1+\xi}{2}}(\lambda-\tau), & \hat{d} &= \sqrt{\frac{1+\xi}{2}}(D-d). \end{aligned} \quad (6.9)$$

With these definitions, the action now reads

$$\begin{aligned} S &= \int d^3x \left(-\frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} - \frac{1}{2} s^\dagger \hat{\square} s + \frac{1}{2} D^2 - \left(\frac{ie_1}{2} (|s|^2 - s_0^2) + \frac{ig_1}{2} (|t|^2 - t_0^2) \right) \hat{D} \right), \\ &\quad - \frac{1}{2} \hat{M} \square \hat{M} - \frac{1}{4} \hat{M}^2 |s|^2 + \frac{i}{2} \bar{\psi} \hat{D} \psi + \frac{i}{2} \bar{\lambda} \hat{\phi} \lambda - \frac{1}{2} \hat{M} \bar{\psi} \psi - \frac{e}{2} (\bar{\psi} \hat{\lambda} s + s^\dagger \bar{\lambda} \psi), \\ &\quad - \frac{1}{4} \hat{G}_{\mu\nu} \hat{G}^{\mu\nu} - \frac{1}{2} t^\dagger \hat{\square} t + \frac{1}{2} d^2 - \left(\frac{ie_2}{2} (|s|^2 - s_0^2) + \frac{ig_2}{2} (|t|^2 - t_0^2) \right) \hat{d}, \\ &\quad - \frac{1}{2} \hat{N} \square \hat{N} - \frac{1}{4} \hat{N}^2 |t|^2 + \frac{i}{2} \bar{\sigma} \hat{D} \sigma + \frac{i}{2} \bar{\tau} \hat{\phi} \tau - \frac{1}{2} \hat{N} \bar{\sigma} \sigma - \frac{g}{2} (\bar{\sigma} \hat{\tau} t + t^\dagger \bar{\tau} \sigma), \end{aligned} \quad (6.10)$$

where the covariant derivatives for the matter sector are now,

$$\hat{D}_\mu = (\partial_\mu - ie_1 \hat{A}_\mu - ie_2 \hat{C}_\mu), \quad \check{D}_\mu = (\partial_\mu - ig_1 \hat{A}_\mu - ig_2 \hat{C}_\mu). \quad (6.11)$$

6.2 Scalar potentials

Since we have not redefined the scalar multiplets at all, solving for the auxiliary fields and restoring the old gauge couplings should give back the previously obtained potential. Let us check this whilst also writing the scalar potential in terms of the new couplings. The absence of the gauge mixing term means that the new auxiliary D -terms are not intertwined and therefore can be eliminated independently;

$$\hat{D} = \frac{ie_1}{2} (|s|^2 - s_0^2) + \frac{ig_1}{2} (|t|^2 - t_0^2) \quad (6.12)$$

$$\hat{d} = \frac{ie_2}{2} (|s|^2 - s_0^2) + \frac{ig_2}{2} (|t|^2 - t_0^2), \quad (6.13)$$

which, after substitution for the couplings given by eq. (6.7) gives rise to the following potential,

$$\begin{aligned} V[s, t] &= \left(\frac{e_1^2 + e_2^2}{8} (|s|^2 - s_0^2)^2 + \frac{g_1^2 + g_2^2}{8} (|t|^2 - t_0^2)^2 + \frac{e_1 g_1 + e_2 g_2}{4} (|s|^2 - s_0^2) (|t|^2 - t_0^2) \right) \\ &= \frac{1}{1 - \xi^2} \left(\frac{e^2}{8} (|s|^2 - s_0^2)^2 + \frac{g^2}{8} (|t|^2 - t_0^2)^2 + \frac{eg\xi}{4} (|s|^2 - s_0^2) (|t|^2 - t_0^2) \right). \end{aligned} \quad (6.14)$$

As expected this expression coincides with the previously obtained scalar potential eq. (2.25).

6.3 Supercharges and algebra

The diagonalisation allows us to easily find the Bogomol'nyi bound for the energy from the simpler supercharge algebra. Indeed, the diagonalized Lagrangian possesses the following supercharges,

$$\begin{aligned}
 Q_S = \int d^2x & \left(\left(-\frac{1}{2}\epsilon^{\mu\nu\rho}\hat{F}_{\mu\nu}\gamma_\rho + i\hat{\phi}\hat{M} + i\hat{D}^* \right) \gamma^0 \hat{\lambda} + \left(i(\hat{\phi} - ie_1\hat{A} - ie_2\hat{C})s^* - \frac{e_1}{2}\hat{M}s^* - \frac{e_2}{2}\hat{N}s^* \right) \gamma^0 \psi \right. \\
 & \left. + \left(-\frac{1}{2}\epsilon^{\mu\nu\rho}\hat{G}_{\mu\nu}\gamma_\rho + i\hat{\phi}\hat{N} + i\hat{d}^* \right) \gamma^0 \hat{\tau} + \left(i(\hat{\phi} + ig_1\hat{A} + ig_2\hat{C})t^* - \frac{g_1}{2}Mt^* - \frac{g_2}{2}Nt^* \right) \gamma^0 \sigma \right), \tag{6.15}
 \end{aligned}$$

and

$$\begin{aligned}
 \bar{Q}_S = \int d^2x & \left(\bar{\lambda}\gamma^0 \left(-\frac{1}{2}\epsilon^{\mu\nu\rho}\hat{F}_{\mu\nu}\gamma_\rho - i\hat{\phi}\hat{M} - i\hat{D} \right) + \bar{\psi}\gamma^0 \left(-i(\hat{\phi} - ie_1\hat{A} - ie_2\hat{C})s - \frac{e_1}{2}\hat{M}s - \frac{e_2}{2}\hat{N}s \right) \right. \\
 & \left. \bar{\tau}\gamma^0 \left(-\frac{1}{2}\epsilon^{\mu\nu\rho}\hat{G}_{\mu\nu}\gamma_\rho - i\hat{\phi}\hat{N} - i\hat{d} \right) + \bar{\sigma}\gamma^0 \left((\hat{\phi} - ig_1\hat{A} - ig_2\hat{C})t - \frac{g_1}{2}Mt - \frac{g_2}{2}Nt \right) \right). \tag{6.16}
 \end{aligned}$$

Note that in this case the conjugate momenta of the fermions are the canonical ones so the procedure is simpler. Calculating the SUSY algebra and imposing that the fermions and gauge scalars vanish, we are left with the following energy and central charge,

$$E = \int d^2x \left\{ \frac{1}{2}\hat{F}_{ij}\hat{F}^{ij} + |\dot{D}s|^2 + \frac{1}{2}\hat{G}_{ij}\hat{G}^{ij} + |\dot{D}t|^2 + V(s, t) \right\}. \tag{6.17}$$

$$\begin{aligned}
 T = - \int d^2x & \left(\epsilon_{ij}\hat{F}^{ij} \left(\frac{e_1}{2}(|s|^2 - s_0^2) + \frac{g_1}{2}(|t|^2 - t_0^2) \right) + i\epsilon^{ij}(\dot{D}_i s)(\dot{D}_j s)^* \right. \\
 & \left. + \epsilon_{ij}\hat{G}^{ij} \left(\frac{e_2}{2}(|s|^2 - s_0^2) + \frac{g_2}{2}(|t|^2 - t_0^2) \right) + i\epsilon^{ij}(\dot{D}_i t)(\dot{D}_j t)^* \right). \tag{6.18}
 \end{aligned}$$

As before this central charge can be written as a total derivative,

$$T = \int d^2x \partial_i \mathcal{V}^i \tag{6.19}$$

with

$$\begin{aligned}
 \mathcal{V}^i = \epsilon^{ij} & \left(\hat{A}_j \left(\frac{e_1}{2}s_0^2 + \frac{g_1}{2}t_0^2 \right) + \frac{i}{2}s^*(\partial_j - ie_1\hat{A}_j - ie_2\hat{C}_j)s \right. \\
 & \left. + \hat{C}_j \left(\frac{e_2}{2}s_0^2 + \frac{g_2}{2}t_0^2 \right) + \frac{i}{2}t^*(\partial_j - ig_1\hat{A}_j - ig_2\hat{C}_j)t \right). \tag{6.20}
 \end{aligned}$$

Then, using Stoke's theorem and imposing that covariant derivatives vanish at infinity in order to have finite energy, we get for the central charge

$$T = \oint dx^i \left((e_1s_0^2 + g_1t_0^2)\hat{A}_i + (e_2s_0^2 + g_2t_0^2)\hat{C}_i \right) = (e_1s_0^2 + g_1t_0^2)\Phi_{\hat{A}} + (e_2s_0^2 + g_2t_0^2)\Phi_{\hat{C}}. \tag{6.21}$$

The energy is thus bounded by a linear combination of the fluxes (proportional to vortex units of flux numbers) of these mixed gauge fields \hat{A}, \hat{C} . We can restore the dependence on the original fields,

$$T = \oint dx^i \left(\left(\frac{e}{2}s_0^2 + \frac{g}{2}t_0^2 \right) (A_i + C_i) + \left(\frac{e}{2}s_0^2 - \frac{g}{2}t_0^2 \right) (A_i - C_i) \right) = es_0^2 \Phi_A + gt_0^2 \Phi_C, \quad (6.22)$$

which is precisely what we found earlier — see eqs. (3.22) and (4.13). For completeness, let us also restore the original fields in the energy,

$$\begin{aligned} E &= \int d^2x \left(\frac{1}{4}(1 - \xi)(F_{ij} + G^{ij})^2 + \frac{1}{4}(1 + \xi)(F_{ij} - G^{ij})^2 + V(s, t) + |D_i^A s|^2 + |D_j^C t|^2 \right) \\ &= \int d^2x \left(\frac{1}{2}F_{ij}F^{ij} + \frac{1}{2}G_{ij}G^{ij} - \xi F^{ij}G_{ij} + V(s, t) + |D_i^A s|^2 + |D_i^C t|^2 \right). \end{aligned} \quad (6.23)$$

Again this is perfectly consistent with the results obtained previously. It is worth noting that the standard Bogomol'nyi approach to finding these quantities by completing various positive terms in the action is, in this circumstance, far more obvious, given that there is no gauge mixing terms. It is indeed, just a question of completing some squares. The positive definite quadratic form that appeared in our previous theory has been diagonalised away.

6.4 Applications of the diagonal theory: correlation functions

Here, in a somewhat unrelated development, we consider a nice implication for the QFT aspects of the gauge kinetic mixed theory as read from the diagonalised theory.

The two theories are exactly equivalent, even at the quantum level since the Jacobian of our transformation is trivial. If we add currents to the partition function, via a term

$$\int d^3x (J_\mu^A A^\mu + J_\mu^C C^\mu) = \int d^3x \begin{pmatrix} J_\mu^A & J_\mu^C \end{pmatrix} \begin{pmatrix} A^\mu \\ C^\mu \end{pmatrix}. \quad (6.24)$$

We retain the shape of this form after diagonalisation as long as we perform the *opposite* transformation on the currents;

$$\begin{pmatrix} J_\mu^A \\ J_\mu^C \end{pmatrix} = \begin{pmatrix} \sqrt{1 - \xi} & 0 \\ 0 & \sqrt{1 + \xi} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} J_\mu^{\hat{A}} \\ J_\mu^{\hat{C}} \end{pmatrix}. \quad (6.25)$$

Thus we can write

$$\frac{\delta}{\delta J_\mu^C} = \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{1 - \xi}} \frac{\delta}{\delta J_\mu^{\hat{A}}} - \frac{1}{\sqrt{1 + \xi}} \frac{\delta}{\delta J_\mu^{\hat{C}}} \right), \quad (6.26)$$

and similarly for the other transformed current. This gives an explicit formula to transform correlation functions in one theory to those in the other theory, thus, we can calculate every observable of one theory from observables of the other.

For instance, looking at the mixing terms diagrammatically, allows us to add a 2-point vertex transforming A into C with amplitude ξ . This is analytically similar to a mass term, in that an arbitrary amount of this vertex can be added to any gauge propagator, which then need to be summed over as a geometric series. In practice: for the $A - A$ and $C - C$

propagator (with the same ingoing and outgoing particle states), one can add any even number of this interaction 2-vertex, leading to a factor ξ^{2n} for each of them, summing over them means that these propagators get modified by a factor of $1/(1 - \xi^2)$. In addition, the Feynman rules now also possess a $A - C$ propagator with different in and out states, corresponding to an odd number of inserted vertices, thus it has a factor of $\xi/(1 - \xi^2)$. This allows for extra channels: one can turn a pair of scalars of one sector into the other scalar pair using this mixed-states propagator.

This is consistent with the results from the diagonalised theory. With these modified gauge propagators, the $ss \rightarrow tt$ tree-level amplitude is proportional to $eg\xi/(1 - \xi^2)$, in the diagonalised theory, summing over both channels we get

$$\frac{eg}{2} \left(\frac{1}{1 - \xi} - \frac{1}{1 + \xi} \right) = \frac{eg\xi}{1 - \xi^2}. \tag{6.27}$$

From this point of view, neither scalar gains an effective charge under the other gauge field. Rather, the gauge field oscillates as it propagates and allows for production of particles in the other sector. Had we performed a non-orthogonal change of basis, such as

$$\tilde{A} = A - \xi C, \quad \tilde{C} = \sqrt{1 - \xi^2} C, \tag{6.28}$$

then s gains (in terms of the new gauge fields) a small hidden sector charge $e\xi/\sqrt{1 - \xi^2}$, while the charge of t is rescaled to $g/\sqrt{1 - \xi^2}$ leading to the same $ss \rightarrow tt$ tree-level amplitude. This approach, while consistent, artificially breaks the equivalence of the two sectors. Indeed, the content of both sectors have the same structure, and are made to communicate by a term that is $A \leftrightarrow C$ invariant, it is more elegant to find an interpretation of this term (i.e. a reformulation of the theory) that does not disturb this property.

Let us now summarize our findings and propose some topics for future investigation.

7 Conclusions

In this paper we have been partly motivated by models for the hidden sector of different mechanisms of SUSY breaking in beyond the Standard Model Physics and also by models with a Higgs portal and gauge kinetic mixing interaction. Our motivation also came from recent developments on Dark Matter and topics around that. Indeed, in some scenarios, the dark-sector is modelled by a lagrangian that communicates it with the Standard Model via a gauge-kinetic mixing interaction. The problem that occupied us in this work was the study of dark-strings, namely topological defects of the dark sector, when in interaction with the visible sector via terms discussed above. Due to different observational constraints, we considered the situation in which both $U(1)$'s — the Standard Model and the hidden one — are spontaneously broken. At large distances this system is well described by two Abelian Higgs models that interact via a gauge kinetic mixing term and a potential to be determined.

We searched for topological objects in this model, using a well established procedure; namely we extended the model of eq. (2.1) to $\mathcal{N} = 2$ SUSY. We then read BPS equations

and topological charges using the SUSY algebra. We have checked our results with more traditional methods, finding complete agreement.

In fact, the $\mathcal{N} = 2$ version of the model of [19] determines the interaction potential and relations between different couplings, so that the model presents stable vortex solutions, generalising those of Nielsen-Olesen. As mentioned, we found the topological charge that bounds the Energy of our strings and BPS equations that control the string dynamics. We have studied these equations numerically, finding the set of parameters that control the shapes and widths of the hidden and visible strings. The relevant parameters are: μ controlling the quotient of the masses of the (spontaneously broken) gauge fields; e_r that controls the VEV of the hidden Higgs field t . Finally, the parameter $\frac{\xi}{e_r}$, accounts for the strength of the interaction.

We observed that in the ‘decoupled’ case $\frac{\xi}{e_r} \rightarrow 0$, for equal masses of the gauge fields $\mu = 1$ and fixed VEV for the hidden Higgs ($e_r \sim 1$), we are in the expected situation of a pair of decoupled Abelian Higgs Models. Changing to $e_r \neq 1$, we found departures from the fully decoupled case; the VEV $\langle t \rangle$ is inversely proportional to e_r , while the profiles of the magnetic fields are still similar. If the masses of the gauge fields are taken to be different (for example $\mu < 1$), the hidden and visible magnetic fields decay differently, the hidden vortex is more delocalised (for $\mu < 1$ and $e_r = 1$). On the other hand, when we consider a ‘strongly mixed’ situation, $\frac{\xi}{e_r} \sim 1$, the profiles of both strings are different, even when the gauge fields acquire the same mass. See figures 1–4 for an illustration of these points.

Finally, we closed our study with a nice alternative way of obtaining these results, by considering a diagonal basis of gauge fields (that on the other hand, charges both hidden and visible matter under both the Standard Model and the hidden gauge groups). Indeed, the gauge kinetic mixing term’s effects is to make the gauge fields oscillate from one sector to another during propagation, this diagonalisation argument is nothing more than a propagation eigenbasis for the theory. Of course, we obtained perfect agreement using different perspectives. It is especially nice that differing formalisms manage to produce a topological (central) charge that is — reasonably — independent of the parameter ξ weighting the gauge kinetic mixing term.

Various problems for future study are suggested by the contents of this paper. Given that the symmetries of the problem reduce it to three space-time dimensions it seems natural to study the behavior when a Chern-Simons term is present — see [28]–[35] — for a sample of different aspects of Chern-Simons SUSY actions and vortex solutions. The study of scattering of visible particles with our topological strings is needed to make concrete predictions on cross sections, that might be experimentally verified. In this sense a more robust numerical analysis than the one presented here would be desirable.

It would also be interesting to study the extension of our formalism, in the case in which the models are non-Abelian (hence applicable when the low Energy description in this paper breaks down) — see [31] for some work on that direction.

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